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Algorithms and technologies for noise analysis in the latent period of an emergency state of control objects

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ABSTRACT

The authors show that the process of the initiation of accidents is reflected in the noisy signals $g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t)$ of appropriate sensors in the form of the noise $\varepsilon_2(t)$. In the process of initiation and development of a defect before it becomes pronounced, the degree of correlation between the noise $\varepsilon_2(t)$ and the sum noise $\varepsilon(i\Delta t) = \varepsilon_1(i\Delta t) + \varepsilon_2(i\Delta t)$, and the useful signal $X(t)$ changes continuously. However, during this period, due to the smallness of the noise $\varepsilon_2(t)$ and to the filtering of the noise $\varepsilon(i\Delta t)$ in the signal $g(t)$, the readings of measuring instruments and the estimates of their statistical characteristics calculated by traditional technologies do not change. The authors propose a technology for forming the noise $\varepsilon^e(t)$ equivalent to the noise $\varepsilon(t)$, a technology for forming the robust equivalent correlation matrices $\vec{R}^e_{gg}$, and correlation and spectral noise technologies for controlling the beginning and dynamics of development of accidents.

1. Introduction

It is known that in real various-purpose control and management systems, the noisy signals $g(t)$ obtained at the outputs of sensors are the sum of the useful signal $X(t)$ and the noise $\varepsilon_1(t)$ [1-3], i.e.

$$g(t) = X(t) + \varepsilon_1(t).$$

The known classical conditions are fulfilled in object's normal state for the centered noisy signals $g(t) = X(t) + \varepsilon_1(t)$ obtained at the outputs of the corresponding sensors, i.e.

$$\left.\begin{aligned} M[X(t)X(t)] \neq 0, M[\varepsilon_1(t)X(t)] = 0 \\ M[X(t)\varepsilon_1(t)] = 0, M[\varepsilon_1(t)\varepsilon_1(t)] \neq 0 \end{aligned}\right\}$$

where M is the mathematical expectation of respective products.

As a result, the formula for calculating the estimate of the variance D_{gg} of the signal $g(t)$ takes the form

$$D_{gg} = M[g(t)g(t)] = M\big[\big(X(t) + \varepsilon_1(t)\big)\big(X(t) + \varepsilon_1(t)\big)\big] = M[X(t)X(t) + X(t)\varepsilon_1(t) + \varepsilon_1(t)X(t) + \varepsilon_1(t)\varepsilon_1(t)] = M[X(t)X(t) + \varepsilon_1(t)\varepsilon_1(t)]$$

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Therefore, we get

$$D_{gg} = M[X(t)X(t) + \varepsilon_1(t)\varepsilon_1(t)] = D_{XX} + D_{\varepsilon_1},$$

where

$$D_{\varepsilon_1} = M[\varepsilon_1(t)\varepsilon_1(t)] = M[\varepsilon(t)\varepsilon(t)] = D_\varepsilon.$$

This happens during the operation of any object when there are inevitable defects from fatigue, wear, corrosion, vibration etc. Starting from that moment, due to the emergence of the noise $\varepsilon_2(t)$, an additional noise appears in the estimate of D_{gg} [3-5]. As a result, the process of the initiation of accidents is reflected in the noisy signal $g(t)$ at the output of the sensor in the form of the noise $\varepsilon_2(t)$. In the process of initiation and development of a defect before it becomes pronounced, the degree of correlation of the noise $\varepsilon_2(t)$ and the useful signal $X(t)$ changes continuously. Therefore, starting from the moment of initiation and development of an accident, the model of the signal $g(t)$ at the sensor output can be represented as

$$g(t) = X(t) + \varepsilon_1(t) + \varepsilon_2(t). \tag{1}$$

The presence of a correlation between the useful signal $X(t)$ and the sum noise $\varepsilon(t) = \varepsilon_1(t) + \varepsilon_2(t)$ results in the inequalities

$$\begin{cases} M[X(t)X(t)] \neq 0, \\ M[X(t)\varepsilon_2(t)] \neq 0, \\ M[\varepsilon_1(t)\varepsilon_1(t)] \neq 0, \\ M[\varepsilon(t)\varepsilon(t)] \neq 0, \\ M[\varepsilon(t)X(t)] \neq 0. \end{cases}$$

and the equalities

$$\begin{cases} M[X(t)\varepsilon_1(t)] = 0, \\ M[\varepsilon_1(t)\varepsilon_2(t)] = 0. \end{cases}$$

Therefore, we have

$$D_{gg} = M\{[X(t) + \varepsilon_1(t) + \varepsilon_2(t)][X(t) + \varepsilon_1(t) + \varepsilon_2(t)]\} = M[X(t)X(t) + X(t)\varepsilon_1(t) + X(t)\varepsilon_2(t) + \varepsilon_1(t)X(t) + \varepsilon_1(t)\varepsilon_1(t) + \varepsilon_1(t)\varepsilon_2(t) + \varepsilon_2(t)X(t) + \varepsilon_2(t)\varepsilon_1(t) + \varepsilon_2(t)\varepsilon_2(t)] = M[X(t)X(t) + X(t)\varepsilon_2(t) + \varepsilon_1(t)\varepsilon_1(t) + \varepsilon_2(t)X(t) + \varepsilon_2(t)\varepsilon_2(t)] = D_{XX} + 2R_{X\varepsilon_2} + D_{\varepsilon_1\varepsilon_1} + D_{\varepsilon_2\varepsilon_2}, \tag{2}$$

where

$$M[X(t)\varepsilon_2(t) + \varepsilon_1(t)\varepsilon_1(t) + \varepsilon_2(t)X(t) + \varepsilon_2(t)\varepsilon_2(t)] = 2R_{X\varepsilon_2} + D_{\varepsilon_1\varepsilon_1} + D_{\varepsilon_2\varepsilon_2} = 2R_{X\varepsilon} + D_{\varepsilon\varepsilon} = D_\varepsilon.$$

Here,

$$D_{\varepsilon\varepsilon} = D_{\varepsilon_1\varepsilon_1} + D_{\varepsilon_2\varepsilon_2}.$$

However, at the beginning of the latent period, due to the smallness of the noise $\varepsilon_2(t)$ in the signal $g(t)$, the readings of measuring instruments do not change. The estimates of the statistical characteristics of the signals $g(t)$ calculated by traditional technologies do not change either. For this reason, the detection of the initiation of accident processes in the latent period T_1 of an emergency state in traditional systems becomes difficult. After the period T_1 ends, the period T_2 begins, when the emergency state becomes pronounced. Then the noise $\varepsilon_2(t)$ becomes commensurable with the noise $\varepsilon_1(t)$, which causes a change in the readings of measuring instruments, as well as in the estimates of D_ε and D_{gg}. For these reasons, the results obtained by traditional technologies, indicating the beginning of object's transition to an emergency state, in some cases turn out to be delayed, which sometimes results in catastrophic accidents [1-5].

It is obvious from expressions (1) and (2) that the estimates of and $D_{\varepsilon_1\varepsilon_2}$ reflect the effects of defect initiation on the sum signal $g(t)$, and therefore the noise $\varepsilon_2(i\Delta t)$ contains the information on the beginning and dynamics of development of the accident. It follows that in order to successfully solve the problem of control of the beginning of accident initiation, it is necessary to extract the information contained in the noise $\varepsilon_2(i\Delta t)$. Therefore, to enhance the reliability and validity of the results of control in the latent period of accident initiation and development, it is advisable to create new effective technologies that allow maximum extraction of information contained in the noise $\varepsilon_2(t)$. Different solutions to this problem are discussed in the following paragraphs.

2. Technology for calculating the approximate values of noise samples

To solve the problem of control and diagnostics in the latent period of object's transition to an emergency state, we shall first of all consider the possibility of calculating the approximate values of the samples of the noise $\varepsilon(i\Delta t)$. Obviously, with this problem solved, it will be possible to correct the readings of measuring instruments and the estimates of the statistical characteristics and to form the equivalent correlation matrices. In view of the above, let us consider one of the possible solutions that comes down to calculating the approximate estimates of the samples of the noise $\varepsilon(i\Delta t)$ that cannot be measured directly. It is clear that if we had the values of estimates of the samples of the noise $\varepsilon(i\Delta t)$ in digital form, we could calculate the noise variance from the following known expression:

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^2\,(i\Delta t). \tag{3}$$

However, it is impossible to directly extract the samples of the noise $\varepsilon(i\Delta t)$ from the samples $g(i\Delta t)$ of the noisy signal. At the same time, using the known expression, we can calculate the estimate of the noise variance D_ε:

$$D_\varepsilon \approx \frac{1}{N}\sum_{i=1}^{N}\left[g^2(i\Delta t) + g(i\Delta t)g\big((i+2)\Delta t\big) - 2g(i\Delta t)g\big((i+1)\Delta t\big)\right]. \tag{4}$$

Obviously, taking into account this expression and introducing the notation

$$g^2(i\Delta t) + g(i\Delta t)g((i+2)\Delta t) - 2g(i\Delta t)g((i+1)\Delta t) = \varepsilon'(i\Delta t),$$

formula (3) for calculating the approximate equivalent values of the samples of the noise $\varepsilon^e(i\Delta t)$ can be written as

$$\begin{aligned}\varepsilon^e(i\Delta t) &= \operatorname{sgn}\varepsilon'(i\Delta t)\\ &\times \sqrt{|g^2(i\Delta t) + g(i\Delta t)g((i+2)\Delta t) - 2g(i\Delta t)g((i+1)\Delta t)|}\\ &= \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}\end{aligned}$$

where $\operatorname{sgn}\varepsilon'(i\Delta t)$ is the sign of the magnitude of $\varepsilon'(i\Delta t)$.

An analysis of possible solutions to this problem has shown [2–5] that it is possible to replace the unmeasurable samples of the noise $\varepsilon(i\Delta t)$ with their approximate equivalent values. For this purpose, it is expedient to use the technology for calculating the estimate of the noise variance D_ε from expression (4), which can also be written as

$$\frac{1}{N}\sum_{i=1}^{N}\varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\left[g(i\Delta t) + g\big((i+2)\Delta t\big) - 2g((i+1)\Delta t\right].$$

As a result, taking the notation

$$\varepsilon'(i\Delta t) = g(i\Delta t)\left[g(i\Delta t) + g\big((i+2)\Delta t\big) - 2g((i+1)\Delta t\right],$$

$$\operatorname{sgn}\varepsilon'(i\Delta t) = \begin{cases} +1 & \text{when} \quad \varepsilon'(i\Delta t) > 0\\ 0 & \text{when} \quad \varepsilon'(i\Delta t) = 0\\ -1 & \text{when} \quad \varepsilon'(i\Delta t) < 0\end{cases}$$

the formula for calculating the equivalent values of the samples of the noise $\varepsilon(i\Delta t)$ can be written as

$$\begin{aligned}\varepsilon(i\Delta t) \approx \varepsilon^e(i\Delta t) &= \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|g(i\Delta t)\left[g(i\Delta t) + g\big((i+2)\Delta t\big) - 2g((i+1)\Delta t\right]\right|}\\ &= \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}.\end{aligned} \tag{5}$$

At the same time, assuming that the following expression is true:

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}{\varepsilon^e}^2(i\Delta t) =$$

$$= \frac{1}{N}\sum_{i=1}^{N} \left| g(i\Delta t)\left[g(i\Delta t) + g\big((i+2)\Delta t\big) - 2g((i+1)\Delta t \right] \right|,$$

the formula for calculating the mean value $\bar{\varepsilon}(i\Delta t)$ of the samples of the noise $\varepsilon(i\Delta t))$ can be reduced to calculating the mean value of the equivalent samples of the noise $\varepsilon^e(i\Delta t)$, i.e.

$$\overline{\varepsilon}(i\Delta t) \approx \overline{\varepsilon^e}(i\Delta t) = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^e(i\Delta t).$$

Numerous experiments have shown that despite the possible deviations of the approximate values of samples $\varepsilon^e(i\Delta t)$ from their true values $\varepsilon(i\Delta t)$ by $\varepsilon^e(i\Delta t) - \varepsilon(i\Delta t)$, the following equality takes place between the estimates of the mathematical expectation and their variance:

$$P\left\{ \frac{1}{N}\sum_{i=1}^{N} {\varepsilon^e}^2(i\Delta t) - \frac{1}{N}\sum_{i=1}^{N} \varepsilon^2(i\Delta t) \approx 0 \right\} = 1,$$

$$P\left\{ \frac{1}{N}\sum_{i=1}^{N} \varepsilon^e(i\Delta t) - \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t) \approx 0 \right\} = 1,$$

where P is the probability sign. These equalities show that by processing $\varepsilon^e(i\Delta t)$ it is possible to get results that would be identical to the results of the analysis of the noise $\varepsilon(i\Delta t)$.

3. Algorithms and technologies for correlation and spectral analysis of the noise

The possibility of calculating the approximate values of equivalent samples of the noise $\varepsilon^e(i\Delta t)$ allows finding the estimates of the statistical characteristics of both the noise and the useful signal from the following expressions:

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} {\varepsilon^e}^2(i\Delta t),$$

$$R_{X\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon(i\Delta t) \approx$$

$$\approx \frac{1}{N}\sum_{i=1}^{N} \left[g(i\Delta t) - \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|} \right] \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|},$$

$$D_x = \frac{1}{N}\sum_{i=1}^{N} X^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} [g(i\Delta t) - \varepsilon^e(i\Delta t)]^2 =$$

$$= \frac{1}{N}\sum_{i=1}^{N} \left[g(i\Delta t) - \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|} \right]^2.$$

With the approximate values of $\varepsilon^e(i\Delta t)$ known, we can calculate the estimates of $R_{X\varepsilon}(\mu)$ and $R_{XX}(\mu)$:

$$R_{X\varepsilon}(\mu) = \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big) \approx$$

$$\approx \frac{1}{N}\sum_{i=1}^{N} \left[g(i\Delta t) - \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|} \right] \operatorname{sgn} \varepsilon'\big((i+\mu)\Delta t\big)\sqrt{\left|\varepsilon'\big((i+\mu)\Delta t\big)\right|},$$

$$R_{XX}(\mu) = \frac{1}{N}\sum_{i=1}^{N} [g(i\Delta t) - \varepsilon^e(i\Delta t)][g(i+\mu)\Delta t - \varepsilon^e((i+\mu)\Delta t)].$$

It is obvious that the equivalent samples of the noise $\varepsilon^e(i\Delta t)$ can also be used to calculate the coefficient of correlation between the noise and the useful signal from the expression, i.e.

$$r_{x\varepsilon} = \frac{R_{X\varepsilon}(0)}{\sqrt{R_{XX}(0)R_{\varepsilon\varepsilon}(0)}} = \frac{R_{X\varepsilon}(0)}{\sqrt{D_X D_\varepsilon}} \approx$$

$$\approx \frac{1}{N}\sum_{i=1}^{N} \left\{ \left[g(i\Delta t) - \operatorname{sgn}[\varepsilon'(i\Delta t))]\sqrt{|\varepsilon'(i\Delta t)|} \right] \right.$$

$$\left. \times \operatorname{sgn}[\varepsilon'(i\Delta t))]\sqrt{|\varepsilon'(i\Delta t)|} \right\}$$

$$\times \left\{ \left[\frac{1}{N}\sum_{i=1}^{N} \left[g(i\Delta t) - \operatorname{sgn}[\varepsilon'(i\Delta t))]\sqrt{|\varepsilon'(i\Delta t)|} \right]^2 \right] \right.$$

$$\left. \times \left[\frac{1}{N}\sum_{i=1}^{N} [\varepsilon'(i\Delta t)] \right] \right\}^{-\frac{1}{2}}.$$

Taking into account expression (5), the algorithms for calculating the spectral coefficients of the noise $\varepsilon(i\Delta t)$ of the noisy signal $g(i\Delta t)$ can be written as

$$a_{n_\varepsilon} = \frac{2}{N}\sum_{i=1}^{N}\varepsilon(i\Delta t)\cos n\omega(i\Delta t) \approx \frac{2}{N}\sum_{i=1}^{N}\varepsilon^{e}(i\Delta t)\cos n\omega(i\Delta t), \quad (6)$$

$$b_{n_\varepsilon} = \frac{2}{N}\sum_{i=1}^{N}\varepsilon(i\Delta t)\sin n\omega(i\Delta t) \approx \frac{2}{N}\sum_{i=1}^{N}\varepsilon^{e}(i\Delta t)\sin n\omega(i\Delta t). \quad (7)$$

Obviously, taking into account (5) – (7), formulas for calculating the estimates of the spectral characteristics of the noise can be written as

$$a_{n_\varepsilon} \approx \frac{2}{N}\sum_{i=1}^{N}\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|g(i\Delta t)\left[g(i\Delta t)+g\big((i+2)\Delta t\big)-2g((i+1)\Delta t\right]\right|}$$
$$\times\cos n\omega(i\Delta t) = \frac{2}{N}\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}\cos n\omega(i\Delta t),$$

$$b_{n_\varepsilon} \approx \frac{2}{N}\sum_{i=1}^{N}\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|g(i\Delta t)\left[g(i\Delta t)+g\big((i+2)\Delta t\big)-2g((i+1)\Delta t\right]\right|}$$
$$\times\sin n\omega(i\Delta t) = \frac{2}{N}\sum_{i=1}^{N}\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}\sin\omega(i\Delta t).$$

4. Technology for forming correlation matrices equivalent to the matrices of useful signals

It is known that when solving problems of identifying the technical condition of facilities in control systems, the formation of correlation matrices of noisy input and output signals is important. The relevance of this problem is due to the fact that the control of the beginning and dynamics of changes in the emergency state of facilities is essential in all areas of industry, power engineering and transport [6-13]. When solving this problem in the normal mode of object's operation, the impulsive admittance function $\vec{W}(\Delta\tau$ is determined on the basis of the measurement of the input signal $X(t)$ and output signal $Y(t)$. To do this, we need to solve a system of linear algebraic equations, which in matrix form is written as follows [1-4]:

$$\vec{R}_{YX} = \vec{R}_{XX}\vec{W}(\Delta\tau), \quad (8)$$

where $\vec{R}_{XX}$ is the square $m\times m$ symmetric matrix of the autocorrelation functions of the centered input signal X(t).

Since in practice signals are distorted by noise, the noisy input $g(t)$ and output $\eta(t)$ signals of the object are the sum of the useful signals $X(t)$ and $Y(t)$ of the corresponding noise ε(t) and φ(t), i.e.

$$g(t) = X(t) + \varepsilon(t),$$
$$\eta(t) = Y(t) + \varphi(t).$$

Then matrix equation (8) can be written in the following form

$$\vec{R}_{\eta g} = \vec{R}_{gg}\vec{W}(\Delta\tau),$$

where

$$\vec{R}_{gg} = \begin{bmatrix} R_{gg}(0) & R_{gg}(\Delta\tau) & \dots & R_{gg}[(m-1)\Delta\tau] \\ R_{gg}(\Delta\tau) & R_{gg}(0) & \dots & R_{gg}[(m-2)\Delta\tau] \\ \dots & \dots & \dots & \dots \\ R_{gg}[(m-1)\Delta\tau] & R_{gg}[(m-2)\Delta\tau] & \dots & R_{gg}(0) \end{bmatrix}, \quad (9)$$

$$\vec{R}_{\eta g} = \begin{bmatrix} R_{\eta g}(0) & R_{\eta g}(\Delta\tau) & \dots & R_{\eta g}[(m-1)\Delta\tau] \end{bmatrix}^{T}.$$

Here,

$$\left.\begin{aligned} R_{gg}(\mu) = M[g(t)g(t+\mu\Delta\tau)] \\ R_{\eta g}(\mu) = M[\eta(t)g(t+\mu\Delta\tau)] \end{aligned}\right\}. \quad (10)$$

In the ideal case, the matrices $\vec{R}_{XX}$ and $\vec{R}_{YX}$ of the correlation functions are formed from the estimates of the correlation functions of the useful signals $X(t)$ and $Y(t)$, which are calculated from the formulas:

$$R_{XX}(\mu) = M[X(t)X(t+\mu\Delta\tau)] = \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)X(i\Delta t + \mu\Delta\tau), \quad (11)$$

$$\vec{R}_{XX} = \begin{bmatrix} R_{XX}(0) & R_{XX}(\Delta\tau) & \dots & R_{XX}[(m-1)\Delta\tau] \\ R_{XX}(\Delta\tau) & R_{XX}(0) & \dots & R_{XX}[(m-2)\Delta\tau] \\ \dots & \dots & \dots & \dots \\ R_{XX}[(m-1)\Delta\tau] & R_{XX}[(m-2)\Delta\tau] & \dots & R_{XX}(0) \end{bmatrix}, \quad (12)$$

$$R_{YX}(\mu) = M[Y(t)X(t+\mu\Delta\tau)] = \frac{1}{N}\sum_{i=1}^{N} Y(i\Delta t)X(i\Delta t + \mu\Delta\tau), \quad (13)$$

$$\vec{R}_{YX} = \begin{bmatrix} R_{YX}(0) & R_{YX}(\Delta\tau) & \dots & R_{YX}[(m-1)\Delta\tau] \\ R_{YX}(\Delta\tau) & R_{YX}(0) & \dots & R_{YX}[(m-2)\Delta\tau] \\ \dots & \dots & \dots & \dots \\ R_{YX}[(m-1)\Delta\tau] & R_{YX}[(m-2)\Delta\tau] & \dots & R_{YX}(0) \end{bmatrix}, \quad (14)$$

$$\vec{R}_{XX} = \begin{bmatrix} R_{XX}(0) & R_{XX}(\Delta\tau) & \dots & R_{XX}[(m-1)\Delta\tau] \end{bmatrix}^T.$$

$$\vec{R}_{YX} = \begin{bmatrix} R_{YX}(0) & R_{YX}(\Delta\tau) & \dots & R_{YX}[(m-1)\Delta\tau] \end{bmatrix}^T.$$

$$\vec{W}(\Delta\tau) = \begin{bmatrix} W(0) & W(\Delta\tau) & \dots & W[(m-1)\Delta\tau] \end{bmatrix}^T.$$

$\vec{W}(\Delta\tau)$ is the column matrix, whose elements are the ordinates of the sought-for impulsive admittance functions.

In practice, correlation matrices (12) and (14) are formed from the estimates $R_{gg}(\mu)$ and $R_{\eta g}(\mu)$ of the correlation functions of the noisy signals $g(t)$ and $\eta(t)$. In this case, the following obvious inequalities arise:

$$\left.\begin{aligned} \vec{R}_{XX} \neq \vec{R}_{gg} \\ \vec{R}_{YX} \neq \vec{R}_{\eta g} \end{aligned}\right\}.$$

As a result, in many cases it is impossible to ensure adequate identification of object's technical condition even under normal operation conditions and if the above-mentioned classical conditions are fulfilled.

The research in [1-4] has demonstrated that the conditions of stationarity and normality of distribution law hold for the input and output noisy signals of many control objects in the normal moder of operation. For this reason, there is no correlation between the useful signals $X(i\Delta t)$ and $Y(i\Delta t)$ and the noises $\varepsilon(i\Delta t)$, $\varphi(i\Delta t)$, i.e.

$$\left.\begin{aligned}\frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big) \approx 0\\ \frac{1}{N}\sum_{i=1}^{N} Y(i\Delta t)\varphi\big((i+\mu)\Delta t\big) \approx 0\\ \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big) \approx 0\\ \frac{1}{N}\sum_{i=1}^{N} \varphi(i\Delta t)\varphi\big((i+\mu)\Delta t\big) \approx 0\\ \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varphi\big((i+\mu)\Delta t\big) \approx 0\end{aligned}\right\} \tag{15}$$

and expressions (11) and (13) for calculating the estimates of the auto- and cross-correlation functions can be written as

$$R_{gg}(\mu) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)g\big((i+\mu)\Delta t\big)$$
$$= \frac{1}{N}\sum_{i=1}^{N}(X(i\Delta t)+\varepsilon(i\Delta t))(X\big((i+\mu)\Delta t\big)+\varepsilon\big((i+\mu)\Delta t\big))$$

$$\approx \begin{cases} R_{XX}(0)+D_\varepsilon & \text{when} \quad \mu = 0;\\ R_{XX}(\mu) & \text{when} \quad \mu \neq 0;\end{cases}$$

$$R_{g\eta}(\mu) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\eta\big((i+\mu)\Delta t\big) = \frac{1}{N}\sum_{i=1}^{N}\big(X(i\Delta t)+\varepsilon(i\Delta t)\big)(Y((i+\mu)\Delta t)$$

$$+\varphi\big((i+\mu)\Delta t)\big) \approx R_{XY}(\mu).$$

Therefore, taking into account that $R_{XX}(0) = R_{gg}(0) - D_\varepsilon$, the correlation matrix $\vec{R}_{gg}(\mu)$, which is equivalent to the matrix $\vec{R}_{XX}(\mu)$ of the useful, signals can be written as follows

$$\vec{R}^{e}_{gg}(\mu) = \begin{Vmatrix} R_{gg}(0)-D_\varepsilon & R_{gg}(\Delta t) & \dots & R_{gg}[(N-1)\Delta t]\\ R_{gg}(\Delta t) & R_{gg}(0)-D_\varepsilon & \dots & R_{gg}[(N-2)\Delta t]\\ \dots & \dots & \dots & \dots\\ R_{gg}[(N-1)\Delta t] & R_{gg}[(N-2)\Delta t] & \dots & R_{gg}(0)-D_\varepsilon \end{Vmatrix}$$

$$\approx \begin{Vmatrix} R_{XX}(0) & R_{XX}(\Delta t) & \dots & R_{XX}[(N-1)\Delta t]\\ R_{XX}(\Delta t) & R_{XX}(0) & \dots & R_{XX}[(N-2)\Delta t]\\ \dots & \dots & \dots & \dots\\ R_{XX}[(N-1)\Delta t] & R_{XX}[(N-2)\Delta t] & \dots & R_{XX}(0) \end{Vmatrix}. \tag{16}$$

Considering that in the absence of correlation between $X(i\Delta t)$, $\varepsilon(i\Delta t)$ and $Y(i\Delta t)$, $\varphi(i\Delta t)$ the following equality takes place

$$R_{g\eta}(0) = R_{XY}(0), \quad R_{g\eta}(\Delta t) = R_{XY}(\Delta t), \dots,$$
$$R_{g\eta}\big((N-1)\Delta t\big) = R_{XY}((N-1)\Delta t),$$

matrix (9) can be represented as the equivalent correlation matrix:

$$\vec{R}^{e}_{g\eta}(\mu) \approx \big[R_{XY}(0) \;\; R_{XY}(\Delta t) \;\; \dots \;\; R_{XY}[(N-1)\Delta t]\big]^{\mathrm{T}} = \vec{R}_{XY}(\mu). \tag{17}$$

Our experimental research has also demonstrated that for objects fulfilling conditions (15), by calculating the estimates of the elements of $R_{g\eta}(\mu)$ from expression (17), it is possible to form the equivalent matrices $\vec{R}^{e}_{g\eta}(\mu)$, whose elements match the elements of the correlation matrix $\vec{R}_{XY}(\mu)$ of the useful signals $X(i\Delta t)$ and $Y(i\Delta t)$.

However, the correlation matrices $\vec{R}_{gg}(\mu)$ of the noisy input signal $g(i\Delta t)$ differs from the correlation matrix $\vec{R}_{XX}(\mu)$ of the useful signal $X(i\Delta t)$ in its diagonal elements, which are the sum of the estimates of the correlation function $R_{XX}(0)$ of the useful signals and the noise variance D_ε. For this reason, in practice, to form the equivalent matrix $\vec{R}^e_{gg}(\mu)$, it is necessary to correct the estimates $R_{gg}(0)$ of its diagonal elements.

For instance, after subtracting the estimate of D_ε from the estimate of the diagonal elements $R_{gg}(0)$ of matrix (16), it can be considered equivalent to matrix (12), i.e.

$$\vec{R}^e_{gg}(\mu) = $$
$$= \left\|\begin{matrix} R_{gg}(0)-D_\varepsilon \approx R_{XX}(0) & R_{gg}(\Delta t)\approx R_{XX}(\Delta t) & \dots & R_{gg}[(N-1)\Delta t]\approx R_{XX}[(N-1)\Delta t] \\ R_{gg}(\Delta t)\approx R_{XX}(\Delta t) & R_{gg}(0)-D_\varepsilon \approx R_{XX}(0) & \dots & R_{gg}[(N-2)\Delta t]\approx R_{XX}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ R_{gg}[(N-1)\Delta t]\approx R_{XX}[(N-1)\Delta t] & R_{gg}[(N-2)\Delta t]\approx R_{XX}[(N-2)\Delta t] & \dots & R_{gg}(0)-D_\varepsilon\approx R_{XX}(0)\end{matrix}\right\|$$

Thus, by calculating the estimates of the noise variances D_ε and D_φ the noisy signals $g(i\Delta t)$ and $\eta(i\Delta t)$, we make it possible to form the matrices $\vec{R}^e_{gg}(\mu)$ and $\vec{R}^e_{g\eta}(\mu)$, which will be equivalent, respectively, to matrices $\vec{R}_{XX}(\mu)$ and $\vec{R}_{XY}(\mu)$ of the useful signals. Therefore, we can assume that the following equalities take place between the matrices of the useful signals and the equivalent matrices of noisy signals:

$$\left.\begin{matrix}\vec{R}^e_{gg}(\mu)\approx \vec{R}_{XX}(\mu) \\ \vec{R}^e_{g\eta}(\mu)\approx \vec{R}_{XY}(\mu)\end{matrix}\right\}.$$

5. Technology for forming the equivalent correlation matrix in the presence between the useful signal and the noise

As noted earlier, it is characteristic of many real-life industrial facilities during operation to go into the latent period of initiation of various defects, such as wear and tear, microcracks, carbon deposition, fatigue strain, etc. [2, 5, 14]. This usually affects the signals received from the corresponding sensors as the noise that in most cases correlates with the useful signals $X(i\Delta t)$ and $Y(i\Delta t)$ [14]. As a result, the sum noise in such cases forms from the noise $\varepsilon_1(i\Delta t)$, which is caused by the external factors, and the noise $\varepsilon_2(i\Delta t)$ that emerges as a result of initiation of various defects. The variance of the noisy signal in that case takes the following form:

$D_g \approx R_{gg}(0) = \frac{1}{N}\sum_{i=1}^{N} g^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} X^2(i\Delta t) + 2\frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon(i\Delta t) + \frac{1}{N}\sum_{i=1}^{N}\varepsilon^2(i\Delta t) \approx R_{XX}(0) + +2R_{X\varepsilon}(0) + D_{\varepsilon\varepsilon}$.

Therefore, when the sum noise

$$\varepsilon(i\Delta t) = \varepsilon_1(i\Delta t) + \varepsilon_2(i\Delta t)$$

has a correlation with the useful signal $X(i\Delta t)$, its variance D_ε is calculated from the expression

$$D_\varepsilon = 2R_{X\varepsilon}(0) + D_{\varepsilon\varepsilon},$$

where $R_{X\varepsilon}(0)$ is the cross-correlation function between the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$; $D_{\varepsilon\varepsilon}$ is the estimate of the variance of the noise $\varepsilon_1(i\Delta t)$.

Therefore, in this case, the variance of the sum noise D_ε is the sum of the variance $D_{\varepsilon\varepsilon}$ of the noise $\varepsilon_1(i\Delta t)$, which is caused by the external factors, and the cross-correlation function $R_{X\varepsilon}(0)$ between the useful signal $X(i\Delta t)$ and the noise $\varepsilon_2(i\Delta t)$, which is caused by the initiation of various processes in the object itself [2, 5, 14]

In view of the above, the formula for calculating the estimate of $R_{gg}(\mu)$ can be represented as follows

$$R_{gg}(\mu) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\, g((i+\mu)\Delta t)$$
$$= \frac{1}{N}\sum_{i=1}^{N}(X(i\Delta t)+\varepsilon(i\Delta t))(X(i+\mu)\Delta t) + \varepsilon((i+\mu)\Delta t))$$
$$= \frac{1}{N}\sum_{i=1}^{N}[X(i\Delta t)\, X\big((i+\mu)\Delta t\big) + \varepsilon(i\Delta t)X\big((i+\mu)\Delta t\big) + X(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big)$$
$$+\varepsilon(i\Delta t)\varepsilon((i+\mu)\Delta t) = R_{XX}(\mu) + R_{\varepsilon X}(\mu) + R_{X\varepsilon}(\mu) + R_{\varepsilon\varepsilon}(\mu)$$

$$\approx \begin{cases} R_{XX}(0) + 2R_{X\varepsilon}(0) + D_{\varepsilon\varepsilon} & \text{when} \quad \mu = 0 \\ R_{XX}(\mu) + 2R_{X\varepsilon}(\mu) & \text{when} \quad \mu \neq 0 \end{cases}.$$

According to our experimental research, in this case, the difficulty of taking account the correlation between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ when forming the correlation matrices is due to the fact that in real-life industrial facilities a correlation between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ often takes place even during several sampling intervals, i.e. when $\mu = \Delta t, 2\Delta t, 3\Delta t, \ldots$ [2, 5, 14].

Therefore, it is necessary to develop a technology for calculating the estimates of $R_{X\varepsilon}(0)$, $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, $R_{X\varepsilon}(3\Delta t)$… of the cross-correlation functions between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$. In this case, by compensating for the errors of the elements $R_{gg} = (0)$, $R_{gg}(\Delta t)$, $R_{gg}(2\Delta t)$, $R_{gg}(3\Delta t)$,… in the corresponding rows and columns of correlation matrices (16), we can ensure that they are equivalent to the matrices of the useful signals. Therefore, to ensure that the correlation matrices are equivalent to the matrices of the useful signals, we need to subtract the values of $2R_{X\varepsilon}(0)$ and $D_{\varepsilon\varepsilon}$ from the estimates of $R_{gg}(0)$, and the value of $2R_{X\varepsilon}(\mu)$ from the estimates of $R_{gg}(\mu)$.

In view of the above, one of the possible ways to correct the errors of the corresponding elements of correlation matrices (9) and (16) is as follows:

$$\vec{R}^{e}_{gg}(\mu) \approx \vec{R}^{e}_{XX}(\mu) \approx$$

$$\approx \begin{Vmatrix} R_{gg}(0) - 2R_{X\varepsilon}(0) - D_{\varepsilon} & R_{gg}(\Delta t) - 2R_{X\varepsilon}(\Delta t) & \ldots & R_{gg}[(N-1)\Delta t] - 2R_{X\varepsilon}[(N-1)\Delta t] \\ R_{gg}(\Delta t) - 2R_{X\varepsilon}(\Delta t) & R_{gg}(0) - 2R_{X\varepsilon}(0) - D_{\varepsilon} & \ldots & R_{gg}[(N-2)\Delta t] - 2R_{X\varepsilon}[(N-2)\Delta t] \\ \ldots & \ldots & \ldots & \ldots \\ R_{gg}[(N-1)\Delta t] - 2R_{X\varepsilon}[(N-1)\Delta t] & R_{gg}[(N-2)\Delta t] - 2R_{X\varepsilon}[(N-2)\Delta t] & \ldots & R_{gg}(0) - 2R_{X\varepsilon}(0) - D_{\varepsilon} \end{Vmatrix}$$

In this case, the correlation matrix of the normalized estimates can be written as

$$\vec{r}^{e}_{gg}(\mu) \approx \vec{r}^{e}_{XX}(\mu) \approx$$

$$\approx \begin{Vmatrix} 1 & \frac{R_{gg}(\Delta t) - 2R_{X\varepsilon}(\Delta t)}{D_g - D_{\varepsilon}} & \ldots & \frac{R_{gg}[(N-1)\Delta t] - 2R_{X\varepsilon}[(N-1)\Delta t]}{D_g - D_{\varepsilon}} \\ \frac{R_{gg}(\Delta t) - 2R_{X\varepsilon}(\Delta t)}{D_g - D_{\varepsilon}} & 1 & \ldots & \frac{R_{gg}[(N-2)\Delta t] - 2R_{X\varepsilon}[(N-2)\Delta t]}{D_g - D_{\varepsilon}} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{R_{gg}[(N-1)\Delta t] - 2R_{X\varepsilon}[(N-1)\Delta t]}{D_g - D_{\varepsilon}} & \frac{R_{gg}[(N-2)\Delta t] - 2R_{X\varepsilon}[(N-2)\Delta t]}{D_g - D_{\varepsilon}} & \ldots & 1 \end{Vmatrix}$$

Our experimental analysis of the noisy signals received at seismic-acoustic stations [15, 16] compressor stations, as well as fixed offshore platforms [17], oil and gas extraction and refining facilities [2, 5, 14], aircrafts [14], as well as of biological signals [18] has demonstrated that a correlation often takes place between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ at different time shifts. The maximum time shift during those experiments did not exceed $\mu = 6\Delta t$, i.e. the correlation disappeared at $\mu = 6\Delta t$. The estimate $R_{X\varepsilon}(m\Delta t)$ can calculated from the generalized expression

$$R_{X\varepsilon}(m\Delta t) \approx \frac{1}{2} R'_{X\varepsilon}(m\Delta t) \approx \frac{1}{2N} \sum_{i=1}^{N} [g(i\Delta t) g\big((i+m)\Delta t\big) - 2g(i\Delta t) g\big((i+m+1)\Delta t\big) + g(i\Delta t) g\big((i+m+2)\Delta t\big)]. \quad (18)$$

where $m = 1,2,3, \ldots$.

Due to this, the use of expression (18) opens the possibility of correcting the corresponding elements of the correlation matrices by calculating the estimates $R_{X\varepsilon}(0)$, $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, $R_{X\varepsilon}(3\Delta t)$, …. For instance, in the presence of a correlation between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ in the elements $R_{gg}(\Delta t)$, $R_{gg}(2\Delta t)$, $R_{gg}(3\Delta t)$, … their estimates are corrected by subtracting from them the corresponding estimates $R_{X\varepsilon}(0)$, $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, $R_{X\varepsilon}(3\Delta t)$, … and the value of D_{ε}. For clarity, demonstrated below is the procedure of correction for the case when $R_{X\varepsilon}(\Delta t) > 0$, $R_{X\varepsilon}(2\Delta t) \approx 0$, $R_{X\varepsilon}(3\Delta t) \approx 0$, …, according to which the element of the second column of the first row and the second row of the first column of the matrix is corrected by the estimate $R_{X\varepsilon}(\Delta t) > 0$:

$$\vec{R}^{e}_{gg}(\mu) \approx \vec{R}^{e}_{XX}(\mu) \approx$$

$$\left\|\begin{matrix} R_{gg}(0)-2R_{X\varepsilon}(0)-D_\varepsilon \approx R_{XX}(0) & R_{gg}(\Delta t)-2R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t) & \dots & R_{gg}[(N-1)\Delta t] \approx R_{XX}[(N-1)\Delta t] \\ R_{gg}(\Delta t)-2R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t) & R_{gg}(0)-2R_{X\varepsilon}(0)-D_\varepsilon \approx R_{XX}(0) & \dots & R_{gg}[(N-2)\Delta t] \approx R_{XX}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ R_{gg}[(N-1)\Delta t] \approx R_{XX}[(N-2)\Delta t] & R_{gg}[(N-2)\Delta t] \approx R_{XX}[(N-2)\Delta t] & \dots & R_{gg}(0)-2R_{X\varepsilon}(0)-D_\varepsilon \approx R_{XX}(0) \end{matrix}\right\|$$

$$\vec{r}^{\,e}_{gg}(\mu) \approx \vec{r}^{\,e}_{XX}(\mu) \approx$$

$$\approx \left\|\begin{matrix} 1 & \frac{R_{gg}(\Delta t)-2R_{X\varepsilon}(\Delta t)}{D_g-D_\varepsilon} \approx R_{XX}(\Delta t) & \dots & \frac{R_{gg}[(N-1)\Delta t]}{D_g-D_\varepsilon} \approx R_{XX}[(N-1)\Delta t] \\ \frac{R_{gg}(\Delta t)-2R_{X\varepsilon}(\Delta t)}{D_g-D_\varepsilon} \approx R_{XX}(\Delta t) & 1 & \dots & \frac{R_{gg}[(N-2)\Delta t]}{D_g-D_\varepsilon} \approx R_{XX}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ \frac{R_{gg}[(N-1)\Delta t]}{D_g-D_\varepsilon} \approx R_{XX}[(N-1)\Delta t] & \frac{R_{gg}[(N-2)\Delta t]}{D_g-D_\varepsilon} \approx R_{XX}[(N-2)\Delta t] & \dots & 1 \end{matrix}\right\|.$$

Obviously, after such correction the obtained matrix can be considered equivalent to the matrix of the useful signals.

Given the importance of solving the problem of identifying object's technical condition with the use of matrix equations, another alternative is proposed below for forming the equivalent correlation matrices. This is carried out by calculating the estimates of the elements of the correlation matrices from the equivalent samples of the useful signal $X^e(i\Delta t)$:

$$X^e(i\Delta t) \approx g(i\Delta t) - \varepsilon^e(i\Delta t),$$

Taking into account formula (5), this expression takes the form:

$$X^e(i\Delta t) \approx g(i\Delta t) - \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{|\,\varepsilon'(i\Delta t)|}.$$

Here we take into account that, in spite of certain errors in the samples $X^e(i\Delta t)$ compared with the samples of the useful signals $X(i\Delta t)$, the following equality holds if the observation time T is long enough:

$$\left.\begin{matrix} P[X_1(i\Delta t) \ge X_1^e(i\Delta t)] = P[X_1(i\Delta t) \le X_1^e(i\Delta t)] \\ P[X_2(i\Delta t) \ge X_2^e(i\Delta t)] = P[X_2(i\Delta t) \le X_2^e(i\Delta t)] \\ \dots\dots\dots \\ P[X_n(i\Delta t) \ge X_n^e(i\Delta t)] = P[X_n(i\Delta t) \le X_n^e(i\Delta t)] \\ P[X_1((i+\mu)\Delta t) \ge X_1^e((i+\mu)\Delta t)] = P[X_1((i+\mu)\Delta t) \le X_1^e((i+\mu)\Delta t)] \\ P[X_2((i+\mu)\Delta t) \ge X_2^e((i+\mu)\Delta t)] = P[X_2((i+\mu)\Delta t) \le X_2^e((i+\mu)\Delta t)] \\ \dots\dots\dots \\ P[X_n((i+\mu)\Delta t) \ge X_n^e((i+\mu)\Delta t)] = P[X_n((i+\mu)\Delta t) \le X_n^e((i+\mu)\Delta t)] \end{matrix}\right\}$$

Due to this, the equivalent correlation matrices can also be formed by extracting approximate values of the samples of the noise $\varepsilon^e(i\Delta t)$ from the sum signal $g(i\Delta t)$. In this case, using the obtained approximate values of the samples of the useful signal $X^e(i\Delta t)$, it is possible to form correlation matrices equivalent to the matrices of the useful signals $X(i\Delta t)$.

To this end, from these approximate values of the samples of the useful signal $X^e(i\Delta t)$, the estimates of the correlation functions $R^e{}_{XX}(\mu)$ and $R^e{}_{XY}(\mu)$ are calculated, using the known technology. Therefore, in this alternative option, it is possible, by extracting the samples of the noise from the noisy signal, to form the equivalent correlation matrices $\vec{R}_{XX}(\mu)$ and $\vec{R}^e{}_{XX}(\mu)$ using the approximate values of the samples of the useful signal $X^e(i\Delta t)$:

$$\vec{R}_{XX}(\mu) =$$

$$= \left|\begin{matrix} \mathrm{M}[X_1(i\Delta t)X_1((i+\mu)\Delta t)] & \mathrm{M}[X_1(i\Delta t)X_2((i+\mu)\Delta t)] & \dots & \mathrm{M}[X_1(i\Delta t)X_n((i+\mu)\Delta t)] \\ \mathrm{M}[X_2(i\Delta t)X_1((i+\mu)\Delta t)] & \mathrm{M}[X_2(i\Delta t)X_2((i+\mu)\Delta t)] & \dots & \mathrm{M}[X_2(i\Delta t)X_n((i+\mu)\Delta t)] \\ \dots & \dots & \dots & \dots \\ \mathrm{M}[X_n(i\Delta t)X_1((i+\mu)\Delta t)] & \mathrm{M}[X_n(i\Delta t)X_2((i+\mu)\Delta t)] & \dots & \mathrm{M}[X_n(i\Delta t)X_n((i+\mu)\Delta t)] \end{matrix}\right|,$$

$$\vec{R}^e{}_{XX}(\mu) =$$

$$= \begin{vmatrix} \mathrm{M}[X^e{}_1(i\Delta t)X^e{}_1((i+\mu)\Delta t)] & \mathrm{M}[X^e{}_1(i\Delta t)X^e{}_2((i+\mu)\Delta t)] & \dots & \mathrm{M}[X^e{}_1(i\Delta t)X^e{}_n((i+\mu)\Delta t)] \\ \mathrm{M}[X^e{}_2(i\Delta t)X^e{}_1((i+\mu)\Delta t)] & \mathrm{M}[X^e{}_2(i\Delta t)X^e{}_2((i+\mu)\Delta t)] & \dots & \mathrm{M}[X^e{}_2(i\Delta t)X^e{}_n((i+\mu)\Delta t)] \\ \dots & \dots & \dots & \dots \\ \mathrm{M}[X^e{}_n(i\Delta t)X^e{}_1((i+\mu)\Delta t)] & \mathrm{M}[X^e{}_n(i\Delta t)X^e{}_2((i+\mu)\Delta t)] & \dots & \mathrm{M}[X^e{}_n(i\Delta t)X^e{}_n((i+\mu)\Delta t)] \end{vmatrix}.$$

The numerous computational experiments have confirmed the validity of the equality

$$\vec{R}_{XX}(\mu) = \vec{R}^e{}_{XX}(\mu).$$

Thus, we can assume that the correlation matrix $\vec{R}^e_{XX}(\mu)$ formed from the estimates calculated from the approximate values of samples of the useful signals $X^e(i\Delta t)$ will be equivalent to the correlation matrix $\vec{R}_{XX}(\mu)$ of the useful signals.

Similar arguments can also be made for the column vectors of the cross-correlation matrices $\vec{R}_{XY}(\mu)$ and $\vec{R}^e_{XY}(\mu)$.

It follows that in the investigated variant, the solution to the problem of control of object's technical condition in the latent period of its emergency state can be reduced to solving the system of matrix equations

$$\vec{R}^e_{XY}(\mu) = \vec{R}^e_{XX}(\mu)\vec{W}(\mu),$$

This opens up new opportunities for solving a wide range of control and identification problems in various fields of technology that require forming the correlation matrices $\vec{R}^e_{XX}(\mu)$ and $\vec{R}^e_{XY}(\mu)$ equivalent to the correlation matrices $\vec{R}_{XX}(\mu)$ and $\vec{R}_{XY}(\mu)$ of the useful signals.

6. Conclusion

Our analysis of the types and stages of initiation and development of defects that precede accidents on technical facilities has shown that the registration of the beginning of the latent period of objects' transition to an emergency state based on the results of traditional technologies of analysis of measurement information in control systems in some cases is delayed due to the inability to analyze the noise correlated with the useful signal. This sometimes results in accidents with catastrophic consequences. The proposed algorithms and noise analysis technologies allow forming corresponding sets of informative attributes to control the beginning of the latent period and the development dynamics of accidents. Their application in the construction of various intelligent noise control systems for oil and gas production facilities, drilling rigs, offshore fixed platforms, compressor stations, in transport, aviation, power engineering, construction, seismology and medicine will improve the safety of their operation, making it more accident-free.

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