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An algorithm for the construction of suboptimistic and subpessimistic solutions of a mixed integer knapsack problem with interval data

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ABSTRACT

In the mixed-integer knapsack problem with interval data the concepts of admissible, optimistic, pessimistic, suboptimistic and subpessimistic solutions are introduced. The algorithms for their construction are developed. The programs of the developed algorithms are compiled and a number of computational experiments on large dimension problems are carried out. These experiments once again confirm the high quality of the developed methods.

1. Introduction

The following problem is considered:

$$\sum_{j=1}^{n}[\underline{c}_j,\overline{c}_j]\,x_j+\sum_{j=n+1}^{N}[\underline{c}_j,\overline{c}_j]\,x_j\to\max \tag{1}$$

$$\sum_{j=1}^{n}[\underline{a}_j,\overline{a}_j]\,x_j+\sum_{j=n+1}^{N}[\underline{a}_j,\overline{a}_j]\,x_j\le[\underline{b},\overline{b}], \tag{2}$$

$$0\le x_j\le d_j\quad \left(j=\overline{1,N}\right), \tag{3}$$

$$x_j,\text{ integers }\left(j=\overline{1,n}\right),(n\le N) \tag{4}$$

Here it is assumed that $0<\underline{c}_j\le\ \overline{c}_j, 0\le\underline{a}_j\le\ \overline{a}_j,\ d_j>0,\left(j=\overline{1,N}\right), 0<\underline{b}\le\overline{b}$ and are integer.

It should be noted that for the coefficients of the constraint (2) the following conditions must be fulfilled.

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$$\sum_{j=1}^{N} \underline{a}_j d_j > \overline{b}.$$

If the condition

$$\sum_{j=1}^{N} \overline{a}_j d_j \leq \underline{b},$$

is fulfilled, then this will not be a constraint.

This problem is said to be an interval mixed integer knapsack problem. It should be noted that problem (1)-(4) is more general than the well-known interval knapsack problem, integer knapsack problem, mixed-integer knapsack problem of linear programming with one constraint, interval problem of linear programming with one constraint. Since for $n = 0$ we obtain an interval problem of linear programming with one constraint, when $n = N$, we obtain a well-known interval knapsack problem, but in the case of $\underline{c}_j = \overline{c}_j$, $\underline{a}_j = \overline{a}_j$, $\left(j = \overline{1, N}\right)$, $\underline{b} = \overline{b}$, we obtain a mixed-integer knapsack problem or an integer knapsack problem. Problem (1)-(4) belongs to the class of NP-complete problems, i.e. difficult solvable since all the particular cases of this problem are NP-complete.

It should be indicated that all classes of problem (1)-(4) were investigated and specific algorithms of solutions for them have been developed in [1-7]. But in [8-10] only an interval problem of linear programming (non-integer) was researched.

In this paper, proceeding from an economic interpretation of the problem (1)-(4), the concepts of admissible, optimistic, pessimistic, suboptimistic and subpessimistic solutions are introduced. Algorithms for construction of suboptimistic and subpessimistic solutions are developed. In the development of these algorithms, the basic principles of interval calculus presented in [11] are used.

Note that in [3 and 5] the interval Boolean knapsack problem was considered. In addition, in [5] methods of constructing suboptimistic and subpessimistic solutions of interval problems were developed, where all variables take values 0 and 1. In this paper of the considered interval problem, some of the variables take integer values, and the rest of the variables change continuously. It is clear that the problem considered in this article is more general. Therefore, the method developed here can be used to solve in particular the interval Boolean knapsack problem. On the contrary, by the method proposed in [3 and 5], it is impossible to solve the considered interval problem of mixed integer programming in this work.

2. Problem statement

First, we introduce some economic interpretation for problem (1)-(4). Suppose an enterprise (company) produces N types of products. n types of these, $(n \leq N)$ should be piece and for $N - n$ types − non-piece. Assume that the consumption for the production of per unit of the j^{th} type of product is in the interval $\left[\underline{a}_j, \overline{a}_j\right], \left(j = \overline{1, N}\right)$. In this case, the profit for per unit of the j^{th} product is included in the interval $\left[\underline{c}_j, \overline{c}_j\right], \left(j = \overline{1, N}\right)$. Assume that for the production of this product a resource is allocated that included in the interval $\left[\underline{b}, \overline{b}\right]$. Obviously, it is necessary to find such a number of products (individually piece and non-piece), for which the total production costs did not exceed the allocated resources included in the interval $\left[\underline{b}, \overline{b}\right]$. Obviously, taking the unknowns $0 \leq x_j \leq d_j$, $\left(j = \overline{1, N}\right)$ and for integers x_j, $\left(j = \overline{1, n}\right)$ we obtain the model (1)-(4).

For problem (1)-(4), we first introduce some definitions that were introduced for problems of mixed Boolean programming with interval data in [6].

Definition 1. $N-$ dimensional vector $\mathrm{X}=(\mathrm{x}_1,\dots,\mathrm{x}_\mathrm{N})$ is called an admissible solution of the problem (1)-(4), if it satisfies system of constraints (2)-(4) for $\forall\, a_j \in [\underline{a}_j,\overline{a}_j]$, $(j=\overline{1,N})$ and $\forall b \in [\underline{b},\overline{b}]$.

Note that, in contrast to the well-known concepts, the concepts of optimal solution and optimal value of the objective function (1) should have a different meaning, since it is impossible to ensure that the sums of intervals of different dimensions do not exceed the fixed ones, and the maximum of the sum of the corresponding intervals in the objective function.

Definition 2. An admissible solution $X^{op}=\left(x_1^{op},x_2^{op},\dots,x_N^{op}\right)$ is called an optimistic solution of the problem (1)-(4), if for $\forall\, b \in [\underline{b},\overline{b}]$, that satisfies the constraint $\sum_{j=1}^{N}\underline{a}_j x_j^{op} \le b$ and in this the value of the function $f^{op}=\sum_{j=1}^{N}\overline{c}_j x_j^{op}$ will be maximal.

Definition 3. An admissible solution $X^{p}=\left(x_1^{p},x_2^{p},\dots,x_N^{p}\right)$ is called a pessimistic solution of problem (1)-(4), if for $\forall b \in [\underline{b},\overline{b}]$ that satisfies the relation $\sum_{j=1}^{N}\overline{a}_j x_j^{p} \le b$ and in this the value of the function $f^{p}=\sum_{j=1}^{N}\underline{c}_j x_j^{p}$ will be maximal.

Since problem (1)-(4) is a generalization of the integer knapsack problem, it is also included in the NP-complete class, i.e. intractable. Therefore, in the following definitions 4 and 5, we have introduced the concepts of suboptimistic and subpessimistic (approximate) solutions.

Definition 4. An admissible solution $X^{so}=(x_1^{so},x_2^{so},\dots,x_N^{so})$ is called an suboptimistic solution of the problem (1)-(4), if for $\forall b \in [\underline{b},\overline{b}]$ that satisfies condition $\sum_{j=1}^{N}\underline{a}_j x_j^{so} \le b$ and in this the value of the function $f^{so}=\sum_{j=1}^{N}\overline{c}_j x_j^{so}$ will take on a large value.

Definition 5. An admissible solution $X^{sp}=\left(x_1^{sp},x_2^{sp},\dots,x_N^{sp}\right)$ is called a subpessimistic solution of the problem (1)-(4), if for $\forall b \in [\underline{b},\overline{b}]$ that satisfies the relation $\sum_{j=1}^{N}\overline{a}_j x_j^{sp} \le b$ and in this the value of the function $f^{sp}=\sum_{j=1}^{N}\underline{c}_j x_j^{sp}$ will take on a large value.

3. Theoretical justification of the method

First of all, it should be noted that in Section 2 one economic interpretation of problem (1)-(4) is presented. On the basis of this interpretation we assume that some j^{th} product is produced. Then the expenses of this product should be included in the interval $[\underline{a}_j,\overline{a}_j]$, $(j=\overline{1,N})$ from the allocated general resource $[\underline{b},\overline{b}]$.Obviously, the profit from the sale of the j^{th} product is included in the interval $[\underline{c}_j,\overline{c}_j]$, $(j=\overline{1,N})$. Then the profit of the selected j^{th} product is $[\underline{c}_j,\overline{c}_j]/[\underline{a}_j,\overline{a}_j]$, $(j=\overline{1,N})$. From this, it follows immediately that it is necessary to produce such a product with the number j_* where the ratio $[\underline{c}_{j_*},\overline{c}_{j_*}]/[\underline{a}_{j_*},\overline{a}_{j_*}]$ will be maximal. It is clear that the number j_* should be determined, in the following way:

$$\min_j \frac{[\underline{c}_j,\overline{c}_j]}{[\underline{a}_j,\overline{a}_j]}=\max_j \frac{\overline{c}_j}{\underline{a}_j}=\frac{\overline{c}_{j_*}}{\underline{a}_{j_*}}$$

or

$$j_*=arg\max_j(\overline{c}_j/\underline{a}_j). \tag{5}$$

It should be noted that using formula (5), the selected number j_* will correspond to definition 4, in other words, this approach corresponds to an optimistic strategy. Based on the pessimistic strategy, we similarly determine the selection criterion for the production of products with the number j_* by definition 5, respectively, as follows:

$$j_*=arg\max_j(\underline{c}_j/\overline{a}_j). \tag{6}$$

Note that formulas (5), (6) can be taken as criteria for selecting the unknowns x_{j_*} to construct suboptimistic and subpessimistic solutions, respectively. However, it is necessary to consider the circumstances in which sets the number j_* is, $j_* \in [1, \dots, n]$ or $j_* \in [n+1, n+2, \dots, N]$. To construct solutions, we will take into account these fundamental circumstances.

Based on the above, we have developed algorithms for constructing sub-optimistic and sub-pessimistic solutions to problem (1)-(4). Numerous computational experiments are conducted.

We use the following notations $I = \{1, \dots, n\}$ and $R = \{n+1, n+2, \dots, N\}$. At the beginning of the process of solution construction, we take $X^{so} = (x_1^{so}, x_2^{so}, \dots, x_N^{so}) = (0,0,\dots,0)$ or $X^{sp} = \left(x_1^{sp}, x_2^{sp}, \dots, x_N^{sp}\right) = (0,0,\dots,0)$. In addition, we denote by S the set of numbers of unknowns to which nonzero values are assigned. Obviously, at the beginning $S = \emptyset$. To construct a suboptimistic solution by criterion

$$j_* = arg \max_{j \in I \cup R} (\overline{c}_j / \underline{a}_j) \tag{7}$$

we consider 2 cases:

1. If $j_* \in I$, then we accept $x_{j_*} = \min\left\{d_{j_*}, \left[(b - \sum_{i \in S} \underline{a}_i x_i)/\underline{a}_{j_*}\right]\right\}$, $S := S \cup \{j_*\}$, $I := I \backslash \{j_*\}$. Here $[z]$ denotes the integer part of the number z. Further, the next number j_* is found with the formula (7).

2. If $j_* \in R$, then we accept $x_{j_*} = \min\left\{d_{j_*}, (b - \sum_{i \in S} \underline{a}_i x_i)/\underline{a}_{j_*}\right\}$, $S := S \cup \{j_*\}$, $R := R \backslash \{j_*\}$.

It is necessary to take into account especially the following options:

If $\min\left\{d_{j_*}, (b - \sum_{i \in S} \underline{a}_i x_i)/\underline{a}_{j_*}\right\} = d_{j_*}$, then the computational process is being continued according to the criteria (7) as above. And if $\min\left\{d_{j_*}, (b - \sum_{i \in S} \underline{a}_i x_i)/\underline{a}_{j_*}\right\} = (b - \sum_{i \in S} \underline{a}_i x_i)/\underline{a}_{j_*}$, $x_j^{so} := 0$ is accepted for all $j \notin S$ and the process to construct a suboptimistic solution is completed. The process is completed, as well as, when $I \cup R = \emptyset$.

An algorithm for the construction of a suboptimistic solution:

Step 1. Input $N, n, \underline{a}_j, \overline{a}_j, \underline{c}_j, \overline{c}_j, d_j, \left(j = \overline{1, N}\right), \underline{b}, \overline{b}$.

Step 2. Accept $I = \{1,2, \dots, n\}$, $R = \{n+1, n+2, \dots, N\}$, $b := \overline{b}$ и $x_j^{so} := 0$, $j \in I \cup R$.

Step 3. Find j_* from the criteria $j_* = arg \max_{j \in I \cup R} (\overline{c}_j / \underline{a}_j)$. If $I \cup R = \emptyset$, then pass to the step 7.

Step 4. If $j_* \in I$, then accept $x_{j_*} = \min\left\{d_{j_*}, \left[(b - \sum_{i \in S} \underline{a}_i x_i)/\underline{a}_{j_*}\right]\right\}$, $S := S \cup \{j_*\}$, $I := I \backslash \{j_*\}$ and pass to the step 3.

Step 5. If $j_* \in R$, then accept $x_{j_*} = \min\left\{d_{j_*}, (b - \sum_{i \in S} \underline{a}_i x_i)/\underline{a}_{j_*}\right\}$, $S := S \cup \{j_*\}$, $R := R \backslash \{j_*\}$. If $x_{j_*} = (b - \sum_{i \in S} \underline{a}_i x_i)/\underline{a}_{j_*}$, then accept $x_j^{so} := 0$ для $j \notin S$ and pass to the step 7, else pass to the step 3.

Step 6. Compute $f^{so} := \sum_{j=1}^{N} \overline{c}_j x_j^{so}$.

Step 7. Print $f^{so}, x^{so} = (x_1^{so}, x_2^{so}, \dots, x_N^{so})$.

Step 8. Stop.

4. Results of computational experiments

To determine the efficiency of the method proposed, the algorithms for constructing of suboptimistic and subpessimistic solutions are compiled under Turbo Pascal, and a number of computational experiments on random problems of various dimensions are carried out. The coefficients of solved problems satisfy the following conditions and are pseudorandom two-digit or three-digit numbers:

I. $1 \le \underline{a}_j \le 99$, $1 \le \overline{a}_j \le 99$, $1 \le \underline{c}_j \le 99$, $1 \le \overline{c}_j \le 99$, $\left(j = \overline{1, N}\right)$.

II. $1 \le \underline{a}_j \le 999$, $1 \le \overline{a}_j \le 999$, $1 \le \underline{c}_j \le 999$, $1 \le \overline{c}_j \le 999$, $\left(j = \overline{1, N}\right)$.

$\underline{b} := [\frac{1}{3}\sum_{j=1}^{N} \underline{a}_j d_j]$, $\overline{b} := [\frac{1}{3}\sum_{j=1}^{N} \overline{a}_j\, d_j]$.

Here $d_j = 10$, $(j = \overline{1, N})$ is accepted.

Table 1

Results of solved problems with two-digit coefficients. ($N = 100; n = 60$)

№	$\underline{b}$	$\overline{b}$	f_m^{so}	f_c^o	f_i^o	δ^o	f_m^{sp}	f_c^p	f_i^p	δ^p
1	25079	25103	51839.42	51839.92	51828	0.00095	30559.21	30559.21	30547	0.00000
2	23756	23976	53901.43	53910.33	53866	0.01651	31235.31	31235.31	31223	0.00000
3	22566	22599	54809.42	54809.42	54784	0.00000	33107.25	33108.17	33088	0.00278
4	23636	23746	53268.28	53268.96	53253	0.00128	29130.31	29131.23	29128	0.00315
5	22906	22939	52532.80	52532.80	52510	0.00000	28861.45	28862.75	28849	0.00449

Table 2

Results of solved problems with two-digit coefficients.($N = 200; n = 100$)

№	$\underline{b}$	$\overline{b}$	f_m^{so}	f_c^o	f_i^o	δ^o	f_m^{sp}	f_c^p	f_i^p	δ^p
1	49169	49319	102951.94	102951.94	102930	0.00000	58825.29	58825.69	58815	0.00069
2	47259	47293	109155.52	109156.52	109149	0.00091	61686.87	61687.15	61672	0.00045
3	47876	47926	100421.27	100421.35	100410	0.00007	59621.05	59621.05	59602	0.00000
4	46709	46743	110786.98	110786.98	110775	0.00000	62201.88	62201.88	62192	0.00000
5	46489	46739	99985.60	99985.60	99981	0.00000	58187.49	58187.49	58184	0.00000

Table 3

Results of solved problems with two-digit coefficients. ($N = 500; n = 300$)

№	$\underline{b}$	$\overline{b}$	f_m^{so}	f_c^o	f_i^o	δ^o	f_m^{sp}	f_c^p	f_i^p	δ^p
1	121666	121699	256929.48	256929.53	256920	0.00002	145745.20	145745.20	145741	0.00000
2	117879	117913	264728.52	264728.52	264728	0.00000	151465.20	151465.20	151448	0.00000
3	119749	119783	259880.16	259880.16	259865	0.00000	156047.10	156047.13	156040	0.00002
4	114643	114663	255827.29	255827.29	255824	0.00000	151895.17	151895.17	151880	0.00000
5	119766	119799	256356.60	256356.60	256349	0.00000	152686.16	152686.16	152672	0.00000

Table 4

Results of solved problems with two-digit coefficients. ($N = 1000; n = 600$)

№	$\underline{b}$	$\overline{b}$	f_m^{so}	f_c^o	f_i^o	δ^o	f_m^{sp}	f_c^p	f_i^p	δ^p
1	236079	236113	526419.29	526419.62	526407	0.00006	307095.43	307095.43	307091	0.00000
2	234836	234869	515673.73	515673.76	515664	0.00000	298003.12	298003.23	297988	0.00004
3	234603	234723	520865.43	520865.44	520855	0.00000	298869.81	298869.87	298864	0.00002
4	234939	234973	516178.26	516178.33	516160	0.00001	304419.53	304419.53	304418	0.00000
5	236426	236639	515603.16	515603.16	515602	0.00000	303237.67	303237.67	303230	0.00000

Table 5

Results of solved problems with three-digit coefficients. ($N = 100; n = 60$)

№	$\underline{b}$	$\overline{b}$	f_m^{so}	f_c^o	f_i^o	δ^o	f_m^{sp}	f_c^p	f_i^p	δ^p
1	234873	235106	472585.34	472585.34	472401	0.00000	308802.22	308802.22	308735	0.00000
2	222593	224776	493224.45	493232.82	492880	0.00170	314759.36	314759.36	314673	0.00000
3	213396	213429	502854.53	502857.16	502838	0.00052	335426.37	335426.37	335226	0.00000
4	222599	223683	486812.63	486824.35	486675	0.00241	296555.14	296555.56	296476	0.00014
5	217356	217389	485469.36	485482.12	485296	0.00263	292004.90	292004.90	291827	0.00000

Table 6

Results of solved problems with three-digit coefficients. ($N = 200; n = 100$)

№	$\underline{b}$	$\overline{b}$	f_m^{so}	f_c^o	f_i^o	δ^o	f_m^{sp}	f_c^p	f_i^p	δ^p
1	466606	468119	945072.19	945074.16	945069	0.00021	598981.41	598981.92	598868	0.00009
2	444633	444666	1003162.57	1003168.80	1002971	0.00062	623143.50	623143.50	623103	0.00000
3	447989	448489	904250.40	904277.61	904136	0.00301	602196.40	602196.40	602191	0.00000
4	444759	444793	1012594.12	1012597.47	1012522	0.00033	625751.43	625756.09	625597	0.00074
5	439113	441619	909568.57	909574.97	909523	0.00070	590958.74	590960.69	590900	0.00033

Table 7

Results of solved problems with three-digit coefficients. ($N = 500; n = 300$)

№	$\underline{b}$	$\overline{b}$	f_m^{so}	f_c^o	f_i^o	δ^o	f_m^{sp}	f_c^p	f_i^p	δ^p
1	1148366	1148399	2359638.44	2359638.44	2359604	0.00000	1471270.25	1471274.35	1471126	0.00028
2	1110933	1110966	2418038.42	2418038.86	2417998	0.00002	1525965.57	1525967.70	1525819	0.00014
3	1127116	1127149	2364044.79	2364044.79	2363967	0.00000	1583365.12	1583366.33	1583334	0.00008
4	1079106	1079313	2328347.63	2328347.63	2328249	0.00000	1537343.54	1537343.54	1537314	0.00000
5	1126003	1126036	2349724.96	2349727.44	2349546	0.00011	1543804.08	1543804.08	1543725	0.00000

Table 8

Results of solved problems with three-digit coefficients. ($N = 1000; n = 600$)

№	$\underline{b}$	$\overline{b}$	f_m^{so}	f_c^o	f_i^o	δ^o	f_m^{sp}	f_c^p	f_i^p	δ^p
1	2228703	2228736	4781655.64	4781659.75	4781481	0.00009	3113348.61	3113349.40	3113280	0.00003
2	2216249	2216283	4718196.79	4718197.28	4718063	0.00001	3020861.48	3020861.53	3020828	0.00000
3	2213056	2214239	4764250.84	4764250.84	4764234	0.00000	3029128.19	3029136.24	3029088	0.00027
4	2212756	2212789	4699264.64	4699264.64	4699162	0.00000	3082745.34	3082746.03	3082729	0.00002
5	2224903	2227053	4688267.00	4688267.00	4688220	0.00000	3072220.04	3072220.04	3072190	0.00000

These experiments once again confirm the high quality of the developed method.

The following designations are used in the tables (1-8):

N – the number of all variables;

n – the number of integer variables;

$\underline{b}, \overline{b}$ – lower and upper bound of an interval in constraint (2);

f_m^{so}, f_m^{sp} – suboptimistic and subpessimistic values of the functional of the mixed-integer problem (1)-(4), respectively;

f_c^o, f_c^p – values of the functional of continued optimistic and pessimistic problems, respectively, i.e. an upper bound of the optimistic and pessimistic values of the functional of the problem (1)-(4), respectively;

f_i^o, f_i^p – values of the functional of continues optimistic and pessimistic problems, respectively, i.e. a lower bound of the optimistic and pessimistic values of the functional of the problem (1)-(4), respectively;

δ^o, δ^p – relative errors (in percents) of the suboptimistic and subpessimistic values of the functional of the problem (1)-(4) respectively, i.e.

$$\delta^o = ((f_c^o - f_m^o)/f_c^o) \times 100 \,, \delta^p = ((f_c^p - f_m^p)/f_c^p) \times 100.$$

5. Conclusions

Based on the tables (1-8), the following conclusions may be drawn. The difference between the suboptimistic and subpessimistic values of problem (1)-(4) obtained by the method in this article from the optimistic and pessimistic values of the functional of problem (1)-(4) is not great. In other words, the relative errors of suboptimistic and subpessimistic values from optimistic and pessimistic values, respectively, vary within a range of 0÷1.017%. If δ^o and δ^p take on the value 0, it means that the corresponding suboptimistic and subpessimistic values are simultaneously optimistic and pessimistic values. These circumstances once again confirm the high efficiency of the proposed method developed in this paper. This increases the usage rate of this method for solving real practical problems.

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