

## Applying methods of the fuzzy sets theory to the problem of assessing hydrocarbon reserves based on geological survey data of a section

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ARTICLE INFO	ABSTRACT
<hr/> <i>Article history:</i> Received 19.10.2020 Received in revised form 02.11.2020 Accepted 20.11.2020 Available online 14.12.2020	<hr/> <i>A fuzzy mathematical model of the problem of assessing the reserves of the investigated section based on the data of physical and geological survey is built. To solve the problem, the membership functions of the considered indicators are described and the arithmetic of fuzzy numbers is applied.</i>
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### 1. Introduction

At present, methods of the fuzzy sets theory finds more and more practical applications, which is due to a deeper understanding of the essence of various quantities [1-3; 4, p.62]. For instance, when assessing the quality of system operation based on survey results, the acceptability of some or other indicators used to be determined by finding its values within certain strictly defined limits, whereas now, when this has become possible, the strictness of the limits is abandoned in favor of their fuzzy description.

The methods of the fuzzy sets theory turned out to be very suitable for the study of one of the problems of developing unconventional hydrocarbon reserves. The essence of the problem is as follows. When assessing reserves based on the data of physical and geophysical survey, the parameters under study are unclear. And the degree of profitability of a geological section is assessed by the combination of these indicators. Thus, the degree of possible profitability of a section can be investigated using elements of fuzzy analysis.

### 2. Problem statement

The development of unconventional hydrocarbon reserves is associated with a number of difficulties. Since the development of the deposit and the production of unconventional hydrocarbons has to be profitable, it is necessary to assess the profitability of the deposit development in advance [5]. This is a complex science-intensive task, not only because unconventional hydrocarbons are usually located at great depths, but also because the criteria for assessing the indicators in fact do not

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have strictly clear descriptions.

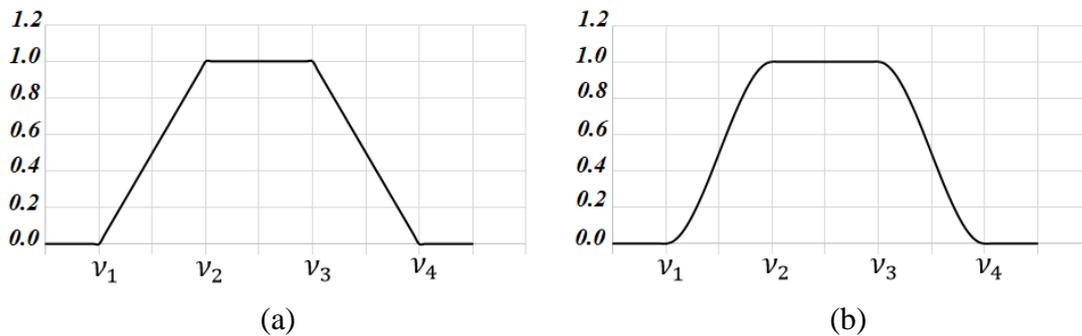
The indicators of the geological section include "Total Organic Carbon", "expansion", "degree of rock maturity", "matrix permeability", "porosity within organic matter", "content of clay fraction", "depth of occurrence", "temperature of the rock", "thickness of the formation".

Each of these indicators has its own measurement limits. For instance, the content of the clay fraction is measured in percentage, it can be in the range from zero to 100% (ideally, if it consists exclusively of clay). However, only the limits in which the section can be considered profitable, e.g.  $(50 \pm 5)\%$ , are meaningful. The unclear nature of the limits manifests itself in the fact that within the limits of measurement accuracy, a section with, for instance, 46% content of the clay fraction, can also be considered as profitable, possibly with some moderate profitability.

Thus, our objective is to describe the criteria for assessing the measurements as a function of membership of the corresponding indicators and, on their basis, to develop a mathematical model of the problem of assessing the profitability of the sections.

**Mathematical formulation.** Let us number the indicators of the section in the order listed above from 1 to 9. In the future, if necessary, instead of the name of the indicator, we will refer to its ordinal number. E.g., the indicator number  $i = 3$  is "degree of rock maturity".

The fuzzy set of profitability by the  $i$ -th parameter is set by the membership function  $\mu_i(x)$ , which takes a value from the interval  $[0, 1]$ . The general types of membership functions for different  $i$  can differ from each other, namely, depending on the nature of the considered indicators, they can have different geometries. To describe these functions, we will use four parameters  $\nu_1, \nu_2, \nu_3, \nu_4$  that determine the "key critical points" of the membership function. Fig. 1 shows two possible variants of membership functions.



**Fig. 1.** Examples of membership functions: (a) – piecewise linear function, (b) – 3rd order piecewise parabolic function.

The geometry function (a) can be analytically represented as follows:

$$\mu(x) = \begin{cases} 0, & x \leq \nu_1, \\ \frac{x-\nu_1}{\nu_2-\nu_1}, & \nu_1 \leq x \leq \nu_2, \\ 1, & \nu_2 \leq x \leq \nu_3, \\ \frac{x-\nu_4}{\nu_3-\nu_4}, & \nu_3 \leq x \leq \nu_4, \\ 0, & \nu_4 \leq x. \end{cases}$$

The geometry function (b) has a more elegant analytical form:

$$\mu(x) = \begin{cases} 0, & x \leq \nu_1, \\ a_1x^3 + b_1x^2 + c_1x + d_1, & \nu_1 \leq x \leq \nu_2, \\ 1, & \nu_2 \leq x \leq \nu_3, \\ a_2x^3 + b_2x^2 + c_2x + d_2, & \nu_3 \leq x \leq \nu_4, \\ 0, & \nu_4 \leq x, \end{cases}$$

where  $a_1 = 2/z_1, b_1 = -3(v_1 + v_2)/z_1, c_1 = 6v_1v_2/z_1, d_1 = -v_1^2(3v_2 - v_1)/z_1, z_1 = (v_1 - v_2)^3, a_2 = 2/z_2, b_2 = -3(v_3 + v_4)/z_2, c_2 = 6v_3v_4/z_2, d_3 = -v_4^2(3v_3 - v_4)/z_1, z_2 = (v_4 - v_3)^3$ .  
 E.g., according to [5], for  $i = 3$  (the "degree of rock maturity" indicator), when selection variant (a) the function has the form

$$\mu(x) = \begin{cases} 0, & x \leq 0.9, \\ 10 \times (x - 0.9), & 0.9 \leq x \leq 1.0, \\ 1, & 1.0 \leq x \leq 1.3, \\ -10 \times (x - 1.4), & 1.3 \leq x \leq 1.4, \\ 0, & 1.4 \leq x, \end{cases}$$

when selecting variant (b) – the form

$$\mu(x) = \begin{cases} 0, & x \leq 0.9, \\ -2000x^3 + 5700x^2 + -5400x + 1701d_1, & 0.9 \leq x \leq 1.0, \\ 1, & 1.0 \leq x \leq 1.3, \\ 2000x^3 - 8100x^2 + 10920x - 4900, & 1.3 \leq x \leq 1.4, \\ 0, & 1.4 \leq x, \end{cases}$$

which corresponds to the following set of parameters:  $v_1 = 0.9, v_2 = 1.0, v_3 = 1.3, v_4 = 1.4$ .

If the set value of the "degree of rock maturity" indicator ( $i = 3$ ) is 0.95, then  $\mu_4(0.95) = 0.5$  means that the degree of possible profitability of the section for such a rock can be estimated at 0.5 or 50% on a 100 point scale.

Thus, it is assumed that all membership functions of the considered indicators  $\mu_i(x)$  are known and measurements of these indicators  $x_i$  of the geological section are given.

The objective is to determine the degree of possible profitability of the section using the apparatus of fuzzy analysis.

### 3. Mathematical model for assessing the profitability of the section

The algorithm for assessing the profitability of the section provides for two options.

The first option (option "1") refers to the case of "Total Organic Carbon" concentration around the value  $x_1 = 1.5$ , while the "expansion" should be 1000 m or more.

The second option (option "2"), when "Total Organic Carbon" is concentrated around the value  $x_1 = 3$ , the acceptable minimum "expansion" is 500 m.

Therefore, for the "Total Organic Carbon" and "expansion" indicators, two options of membership functions will be proposed. Depending on the considered option  $j$  ( $j = 1, 2$ ), the membership functions are denoted by  $\mu_{1,j}(x_1)$  and  $\mu_{2,j}(x_2)$ , respectively. Following the criteria placed by modern geology on various indicators, we present a table of parameters for various membership functions (Table 1). The spread in the values of the parameters is due to the difference in the units of measurement of the values of  $x_i$  ( $i = 1, 2, \dots, 9$ ).

**Table 1**  
Parameters of membership functions

Indicator, [unit of measurement]	Notation	$v_1$	$v_2$	$v_3$	$v_4$
Total Organic Carbon ("1"), [%]	$\mu_{1,1}(x_1)$	0.5	1.1	1.9	2.5
Total Organic Carbon ("2"), [%]	$\mu_{1,2}(x_1)$	1.5	2.1	3.9	4.5
Expansion ("1"), [m]	$\mu_{2,1}(x_2)$	980	1000	9000	9100
Expansion ("2"), [m]	$\mu_{2,2}(x_2)$	98	100	9000	9100

Degree of rock maturity, [%]	$\mu_3(x_3)$	0.9	1.0	1.3	1.4
Matrix permeability, [ $\mu\text{m}^2$ ]	$\mu_4(x_4)$	0.7	0.8	1.2	1.5
Porosity within organic matter, [%]	$\mu_5(x_5)$	4	5	50	55
Content of clay fraction, [%]	$\mu_6(x_6)$	41	48	52	57
Depth of occurrence, [m]	$\mu_7(x_7)$	2900	3000	9000	9100
Temperature of the rock, [ $^{\circ}\text{C}$ ]	$\mu_8(x_8)$	75	75	85	90
Thickness of the formation, [m]	$\mu_9(x_9)$	27	30	1000	1010

For one selected option, the level of possible profitability is determined by the level of satisfaction of all measured indicators, i.e. for the  $j$ -th option ( $j = 1,2$ ), the degree of possible profitability will be calculated from the formula [4, p. 7]:

$$\mu_j(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = \min\{\mu_{1,j}(x_1), \mu_{2,j}(x_2), \mu_3(x_3), \mu_4(x_4), \mu_5(x_5), \mu_6(x_6), \mu_7(x_7), \mu_8(x_8), \mu_9(x_9)\}. \quad (1)$$

Then the assessment of the degree of possible profitability of the section  $\mu$  according to the measurements  $x_i$  ( $i = 1,2, \dots, 9$ ) according to the set of options  $j = 1,2$  will be determined as follows [4, p.7]:

$$\mu(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = \max_{j=1,2}\{\mu_j(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9)\}. \quad (2)$$

System (1)-(2) is a fuzzy mathematical model of the problem of assessing the section reserves based on the data of physical and geological surveys.

There are two important points to note.

1. When studying rock samples in different laboratories, different average values are obtained for the same indicators. Obviously, in this case, various estimates of the section reserves can be obtained, and which of them is more plausible will be decided by the intuition of specialists,

2. Often, due to limited laboratory capacity, it is not always possible to collect data for all indicators. In this case, the missing data for indirect reasons are considered satisfactory and the assessment is made on the basis of the available data.

Let us give an example of calculating unconventional hydrocarbon reserves of the territory of the Yevlakh-Agjabedi trough [5, p.17] by formulas (1)-(2) taking into account the above remarks. Considering that  $x_1 \approx 1.41$ ,  $x_3 \approx 0.96$ ,  $x_4 \approx 0.85$ ,  $x_7 \approx 4500$ ,  $x_9 \approx 150$ , we obtain

$$\mu_1(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = \min\{1.0, 1.0, 0.6, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0\} = 0.6,$$

$$\mu_2(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = \min\{0.0, 1.0, 0.6, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0\} = 0.0,$$

$$\mu(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = \max\{0.6, 0.0\} = 0.6.$$

Thus, the degree of possible profitability of the territory under consideration is 0.6, i.e. from the perspective of prospecting for shale hydrocarbons, this territory can be considered promising.

#### 4. Conclusion

The generally accepted expert estimates of the parameters of shale sections have been described as membership functions. The values of these functions have been interpreted as the level of profitability. For a preliminary assessment of hydrocarbon reserves in shale sections, fuzzy estimates based on laboratory data from field studies can be applied. Based on the proposed approach, a software module has been created for calculating the profitability of shale deposits.

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