Informatics and Control Problems 41 Issue 1 (2021)
journal homepage: www.icp.az

Mixed test method for improving the accuracy of differential measurement systems

M.M. Isayev*, M.B. Mammadova, N.Kh. Badalova

Institute of Control Systems of Azerbaijan National Academy of Sciences, Baku, Azerbaijan

A R T I C L E I N F O

Article history:
Received 15.09.2020
Received in revised form 05.10.2020
Accepted 11.10.2020
Available online 20.05.2021

Keywords:
Measuring instrument
Conversion function
Nonlinear identification
Measurement with hybrid tests
Measurement error correction

A B S T R A C T

Improving the accuracy of measurement systems with non-linear conversion function, identification of their conversion characteristics and test algorithms for measurement error correction are developed. To ensure the regularity of the measurement results, part-by-part nonlinear approximation and differential measurements are applied to the entire measurement range, optimal test values and hybrid sets are determined, basic test equations are set up, and a mathematical model of the conversion characteristics of the measurement system is obtained. The structures of measurement by tests systems are developed, real tests are conducted to confirm the adequacy of the algorithm, and the components of the measurement result error are evaluated.

1. Introduction

The main issue in modern industry is enhancing complex automation and measurement accuracy of technological parameters to improve the efficiency of technological processes and production capacity. Modern measurement systems used in this field must have an optimal and universal structure, intelligent electronic modules, high-precision primary transmitters, efficient measurement algorithms, perfect primary data processing methods, reliable interface circuits.

It is known that primary measuring instruments cannot maintain the stability of their metrological characteristics in real operation for known and unknown reasons. At the same time, if we take into account that their conversion characteristics (CC) are non-linear, then we see that high measurement accuracy is not ensured, and the measurement results are accompanied by fairly large value errors [1-3]. These shortcomings are observed in almost all types of measuring instruments, and for this reason a high-precision identification of their CC is required.

In contrast to the known algorithmic-test methods, differential measurement and hybrid tests are applied here, the coefficients of the conversion function (CF) are replaced with simple additive and multiplicative tests. Here, the accuracy of identification of the CF will depend on the accuracy class of the applied tests, the determined optimal values of simple additive and multiplicative tests and the choice of their corresponding combinations. The main advantages of the proposed measurement method are the simultaneous connection of the measurement quantity and the reference test signals to the inputs of the measuring device, non-linear identification of the CC, determination

*Corresponding author.
E-mail addresses: mezahir@bk.ru (M.M. Isayev), mehriban_m_1983@mail.ru (M.B. Mammadova), bnatige@mail.ru (N.Kh. Badalova).

www.icp.az/2021/1-09.pdf
2664-2085/ © 2021 Institute of Control Systems of ANAS. All rights reserved.

of the largest inflection points, approximation intervals taken in the form of square trinomials, selection of optimal values of simple tests.

The paper gives several functional diagrams of the differential measurement system, appropriate hybrid test algorithms to implement the division of the initial information collected from the measuring instruments into informative and non-informative component, their separate high-precision evaluation.

2. Problem statement – factors affecting the measurement accuracy

Because the input quantities of measurement systems represent a random function of time, they are subject to additive and multiplicative changes in real operation due to the external environment and other random impacts. It is important to develop corrective test algorithms to detect, evaluate and compensate for these impact factors.

All the specified parameters that characterize the input quantity $x(t)$ change over time and are characterized by their own frequency spectra, respectively. Reflecting the dynamics of variation of the quantity $x(t)$ in time, the frequency spectrum has a sufficient effect to obtain a minimum measurement error, which in turn determines the measurement method that can be implemented in the measurement system (MS).

The mathematical dependence between the input and output quantities of the MS is expressed by a conversion function and is generally represented as follows [3]

$$y = F\left(x, \vec{q}, \vec{M}, \vec{T}\right), \tag{1}$$

where $\vec{q}$ is the coefficients to be taken into account during the variation of the output signal of the MS depending on the non-informative parameters of the input signal; $\vec{M}$ is the coefficients to be taken into account during the variation of the parameters of the CF of the MS, depending on the change in the external environment in real operation; $\vec{T}$ is the coefficients to be taken into account during the variation of the random variation of the parameters of the CF of the MS.

In general, since the parameters of the real CF of the MS are non-stationary random time functions, the error of the MS will also be a non-stationary random time function. Then, the measurement error for an arbitrary MS can be shown as follows:

$$\Delta y(t) = \bar{\Delta} y(t) + \dot{\Delta} y(t), \tag{2}$$

where $\bar{\Delta} y(t)$ is the non-stationary random function component; $\dot{\Delta} y(t)$ is the time invariant, centralized, ergodic component.

If we note two sought-for intersections of these functions at the time instants t_1 and t_2, (here $t_2 - t_1 = T$ is the time required to implement all additional transformations provided by the measurement algorithm), then the correlation coefficient $t_{\bar{\Delta}}(t_2, t_1)$ of the random quantities $\bar{\Delta} y(t_2)$ and $\bar{\Delta} y(t_1)$ approaches unit, and the correlation coefficient of the random quantities $\dot{\Delta} y(t_2)$ and $\dot{\Delta} y(t_1)$ approaches zero.

Writing $\Delta y(t)$ as (2) allows dividing the measurement error into two components depending on the frequency spectrum:

1) the autocorrelation component $\bar{\Delta} y(t)$ of the error $\Delta y(t)$, combining random errors that are systematic, increasing, and gradually changing with respect to the time *T*;

2) the short-term autocorrelation component $\dot{\Delta} y(t)$ of the error $\Delta y(t)$ – to the non-autocorrelation component of the error, combining random non-correlated errors of the "white noise" type.

Subsequent stages in the investigation of the error of the tested MS are carried out on the basis of the analysis of the measurement results (MR) obtained as a random time function. Therefore, correlated and non-correlated random quantities will mean autocorrelated and non-autocorrelated random quantities of MR errors at given time instants.

It should be noted that the existing methods of increasing the measurement accuracy allow reducing one of the two components of the above-mentioned MR error. In each case, the effectiveness of some or other method depends on the ratio of the correlated and non-correlated components of the MR error.

3. Problem solution – a test algorithm for improving the measurement accuracy

To improve the accuracy of the initial MS, take a look at the mathematical model of their CF. It is known that the mathematical model of the CF of the MS is generally assumed as follows [3]:

$$y = \sum_{i=1}^{n} a_i x^{i-1}, \quad i = 1,..,n \tag{3}$$

Since the effect of the parameters a_i of the CF in the form of (3) on the measurement accuracy is the main source of error, it is important to eliminate or compensate for it. To do this, additional measurements of n tests $A_i(x), i = x, \dots n$ must be carried out in the MS. In this case, a system of $n + 1$ equations is formed in accordance with the $n + 1$ unknowns $x, a_1, \dots, a_n$ [2]:

$$\begin{cases} y_0 = \sum_{i=1}^{n} a_i x^{i-1} \\ y_1 = \sum_{i=1}^{n} a_i [A_1(x)]^{i-1} \\ y_2 = \sum_{i=1}^{n} a_i [A_2(x)]^{i-1}, \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ y_n = \sum_{i=1}^{n} a_i [A_n(x)]^{i-1} \end{cases} \tag{4}$$

where $y_0, \dots y_n$ are the output parameters of the MS; a_i is the coefficients of CF; $A_n(x)$ is the set of tests together with the input quantity of the OS.

System of equations (4) is a system of linear equations with respect to the parameters $a_1, \dots, a_n$, and the coefficients are found by solving it by Cramer's rule [4, 5]. In the case under investigation, the measurement process is much more complicated due to the need to perform $n + 1$ measurement operations and is considered unsatisfactory as it requires too much extra time.

Depending on the nonlinearity of the CF of the initial MS, the number of tests used in the MS, and their quality ratio, the measurement algorithm will be quite complex for the sought-for quantity x. At the same time, the combined implementation of additive, multiplicative and hybrid tests complicates the measurement process. Taking all this into account, it was confirmed that each interval is selected in the form of square trinomial during part-by-part nonlinear approximation of CF [2, 3].

$$y = a_1 + a_2 x + a_3 x^2, \tag{5}$$

If we take CF in the form of (5), the number of basic test equations (BTE) used in the tested measurement process will be minimized.

The adequacy of the identification of CF depends on the use of one or another type of tests, as well as the obtained accuracy of the additive (θ) and multiplicative (k) fixed test complexes that make them up. In other words, the accuracy of the MR is determined by the accuracy and optimal combination of tests that form the system of equations applied in the tested MS.

Given the above, we obtain the following system of simple equations from system of equations (4):

$$\begin{cases} y_1 = a_{1s} + a_{2s} x_1 + a_{3s} x_1^2 \\ y_2 = a_{1s} + a_{2s} x_2 + a_{3s} x_2^2, \\ y_3 = a_{1s} + a_{2s} x_3 + a_{3s} x_3^2 \end{cases} \tag{6}$$

where x_1, x_2 and x_3 are the measured quantities; y_1, y_2 and y_3 are the output quantities; *s=1,...,i* are the approximation intervals.

The block diagram of the MS implemented with additive tests will look as follows:

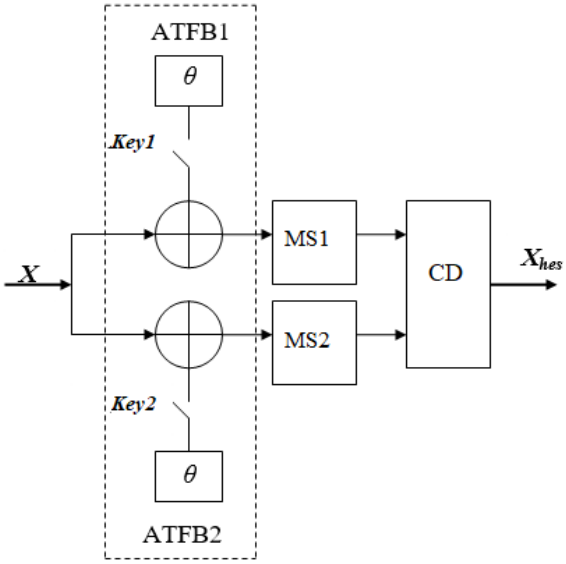

Fig. 1. Block diagram of the tested differential MS

Here, ATFB is the additive test formation block; MS1 and MS2 are the respective measurement systems; CD is the computing device.

In Fig. 1, the measurement process is carried out by simple additive tests in the form $x \pm \theta$ within a given accuracy limit of the measurement quantity x, and they express the additive fixed complexes having the same physical nature as x as follows:

$$\begin{cases} y_0 = a_{1s} + a_{2s}x + a_{3s}x^2 \\ y_1 = a_{1s} + a_{2s}(-x) + a_{3s}(-x)^2 \\ y_2 = a_{1s} + a_{2s}(x+\theta) + a_{3s}(x+\theta)^2 \\ y_3 = a_{1s} - a_{2s}(x+\theta) + a_{3s}[-(x+\theta)]^2 \end{cases} \quad (7)$$

As can be seen from expression (7), the measurement will consist of four cycles: in the first two cycles, MS1 and MS2 tests x and $-x$, and in the next two cycles, tests $(x + \theta)$ and $-(x + \theta)$ are measured. Solving system of test equations (7), we obtain the following expression for the measured quantity x:

$$x = \frac{y_0 - y_1}{(y_2 - y_3) - (y_0 - y_1)}\theta, \quad (8)$$

Thus, (8) represents the CF of the tested MS, and as can be seen from this expression, the accuracy of the measured quantity x depends not on the coefficients of the CF, but on the accuracy of the applied tests.

Using the optimal combination of multiplicative and combination tests — hybrid tests — in addition to additive tests for non-linear CF allows obtaining higher measurement accuracy. The differential measurement diagram of the system based on hybrid tests is given in Fig. 2 below:

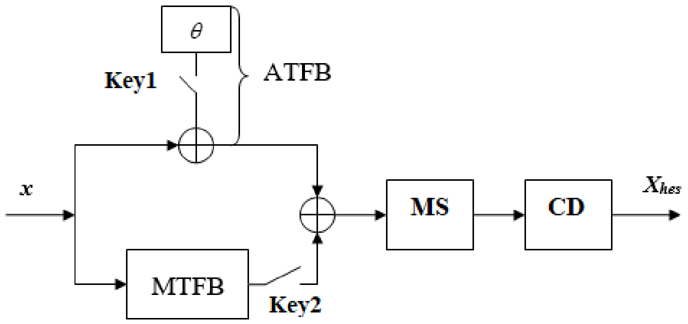

Fig. 2. Block diagram of the MS based on hybrid tests

Here, MTFB is the multiplicative test formation block.

The BTE implemented in this diagram will be as follows:

$$\begin{cases} y_0 = a_{1s} + a_{2s}x + a_{3s}x^2 \\ y_1 = a_{1s} + a_{2s}(x+\theta) + a_{3s}(x+\theta)^2 \\ y_2 = a_{1s} + a_{2s}kx + a_{3s}(kx)^2 \\ y_3 = a_{1s} + a_{2s}(kx+\theta) + a_{3s}(kx+\theta)^2 \end{cases} \tag{9}$$

As can be seen from expression (9), the measurement will consist of four cycles: in the first cycle x, in the second cycle θ, in the third cycle test kx and in the fourth cycle test $kx+\theta$ are measured. Solving system of test equations (9), we obtain the following expression for the CF:

$$y_0 = (y_1 - y_2)\frac{[x(k-1)+\theta](y_1-y_2)}{[x(k-1)-\theta]} + y_3, \tag{10}$$

From (10), we obtain the following expression for the measured quantity x:

$$x = \frac{(y_0+y_1)-(y_2+y_3)}{(y_0-y_1)+(y_2-y_3)} \cdot \frac{\theta}{k-1}, \tag{11}$$

The adequacy of expressions (8) and (11) obtained for the tested MS is determined by the estimation of the resulting measurement error, with the determination of each component.

Thus, for the implementation of the hybrid-test algorithm for increasing the measurement accuracy in the initial MS combined measurements of the tests $x+\theta$; kx and $kx+\theta$ are performed and BTE for the tested MS are compiled based on the optimal combination of these tests. Since reducing the degrees of these equations determines the approximation step or identification accuracy, an estimation of the measurement results is presented below.

4. Algorithm for estimating the static measurement error

Since the real current values of the parameters a_{1s} in each approximation interval differ from their nominal values a_{1sN}, if we consider this difference as an error, then the results of the cycles of measurement of the quantities x, $x+\theta$; kx and $kx+\theta$ will express the errors gaining significant value, and if we consider these errors in system of equations (9), we obtain the following system of equations:

$$\begin{cases} y_0 + \Delta_0 = a_{1s} + a_{2s}x + a_{3s}x^2 \\ y_1 + \Delta_1 = a_{1s} + a_{2s}(x+\theta) + a_{3s}(x+\theta)^2 \\ y_2 + \Delta_2 = a_{1s} + a_{2s}kx + a_{3s}(kx)^2 \\ y_3 + \Delta_3 = a_{1s} + a_{2s}(kx+\theta) + a_{3s}(kx+\theta)^2 \end{cases}, \tag{12}$$

where $\Delta_0, \dots \Delta_3$ are the errors of the measurement cycle received on the outputs of the MS.

Solving system of equations (12), we obtain the following CF for the tested MS:

$$y_0 = \frac{y_1[x(k-1)+\theta]+\Delta_1[x(k-1)+\theta]-y_2[x(k-1)+\theta]-\Delta_2[x(k-1)+\theta]+(y_3+\Delta_3)[x(k-1)-\theta]}{[x(k-1)-\theta]+\Delta_0[x(k-1)-\theta]}, \quad (13)$$

Unlike formula (9), (12) takes into account the errors of the measurement cycles, and their difference will give the error of the basic test equations of the tested MS:

$$\Delta_{BTE}= [x(k-1)+\theta](\Delta_1-\Delta_2)+[x(k-1)-\theta](\Delta_3-\Delta_0), \quad (14)$$

Since (14) is the expression that determines the final error of the tested MS, if we group the values of the tests θ and k, as well as the values of the errors $\Delta_0, \dots \Delta_3$, we obtain a mathematical model of the error components. Then formula (14) will be as follows:

$$\Delta_{BTE}= \theta[(\Delta_1-\Delta_2)-(\Delta_3-\Delta_0)]+x(k-1)[(\Delta_1-\Delta_2)+(\Delta_3-\Delta_0)], \quad (15)$$

Thus, we obtain the following formula for the absolute error Δ_{in}brought to the input of the tested MS:

$$\Delta_{in}= f_{BTE}^{-1}[f_{BTE}(x)+\Delta_{\text{ə}TT}]-x=\frac{\Delta_{BTE}}{f'_{BTE}(x)}, \quad (16)$$

here, $f_{BTE}(x)=(y_3-y_0)[x(k-1)-\theta]+(y_2-y_1)[x(k-1)+\theta]$.

Differentiating formula (13) with respect to x, we obtain the following value of $f'_{BTE}(x)$:

$$f'_{BTE}(x)=(k-1)[(y_0-y_3)-(y_1-y_2)] \quad (17)$$

By substituting (15) and (17) in (16), we obtain the following expression for the input error:

$$\Delta_{in}= \frac{\theta[(\Delta_1-\Delta_2)-(\Delta_3-\Delta_0)]+x(k-1)[(\Delta_1-\Delta_2)+(\Delta_3-\Delta_0)]}{2\theta(1-k)\{a_{2sN}+a_{3sN}[x(k+1)+\theta]\}}, \quad (18)$$

As mentioned above, the errors Δ_i of the measurement cycles are the result of a large number of random factors. Therefore, if the components of the errors Δ_i have the same order in terms of weight coefficients, then according to the central limit theorem, it is possible to put forward the idea that the errors Δ_i obey the law of normal distribution [1, 4].

Analyzing expressions (14) and (18), we can note the following main properties of the errors of MS implemented based on hybrid tests:

1. If the absolute errors Δ_i of the measurement cycles have the same mathematical expectations as the implementation of the obtained algorithms, then the mathematical expectations of the errors of the tested MS operating on these algorithms will be zero.

2. If the conditions between the given tests are met and the additive fixed components of the used tests approach an infinitesimal value, then the absolute error brought to the input of the MS will be approaching infinity.

This property of the error of the tested MS is derived directly from expression (18).

Thus, it can be concluded that the main properties of the errors of the tested MS, in which the algorithms for increasing the measurement accuracy based on simple additive and multiplicative tests are implemented, are justified for the MS operating on the basis of an optimal combination of additive, multiplicative and combination tests.

For the variances of the errors of the tested MS in which the developed algorithms are implemented, the following estimates are obtained accordingly, if the errors of the measurement cycles are independent:

$$\sigma^2_{\Delta_{BTE}} = \sigma^2_{\Delta_0}[x(k-1)-\theta]^2+\sigma^2_{\Delta_1}[x(k-1)+\theta]^2+\sigma^2_{\Delta_2}[x(k-1)+\theta]^2+\sigma^2_{\Delta_3}[x(k-1)-\theta]^2 \quad (19)$$

$$\sigma^2_{\Delta_{BTE\text{ə}TT}} = \left(\sigma^2_{\Delta_0}+\sigma^2_{\Delta_3}\right)(z-\theta)^2+\left(\sigma^2_{\Delta_3}+\sigma^2_{\Delta_2}\right)(z+\theta)^2+4z^2(\sigma^2_{\Delta_1}+\sigma^2_{\Delta_4}), \quad (20)$$

where $\sigma^2_{\Delta_1}$ is the the standard deviation of the measurement error in the corresponding cycles; $z = x(k-1)$ -dir.

As can be seen from expressions (19) and (20), the variance of the error Δ_{BTE} of the BTE of the MS, in which the test algorithm for increasing the measurement accuracy is implemented, is stronger than the variance of single-cycle measurement.

The implementation of the hyperbolic CF and the error estimation can be carried out in a similar way, and in this case the methodology for analyzing the errors of the MR remains unchanged.

This study of the metrological characteristics of the tested MS shows that the following components are the most affected by the MR error of such systems:

– error component caused by the implementation of fixed components of additive and multiplicative tests;

– uncorrelated component of the static error of MS;

– component of the dynamic error of MS;

– error component caused by the inadequacy of the accepted mathematical model of the real CF of the initial MS.

It should be noted that it is possible to organize measurement operations in the same order by increasing the degree of approximation polynomial. However, our numerous studies have shown that it is probably sufficient to take this approximating curve in the form of a quadratic trinomial. Thus, there is no need to take into account these terms, as the coefficients of cubic and subsequent degree terms are very small (close to zero).

The results of differential measurements and statistical evaluation of real tests performed using the developed test algorithms are presented in the table below.

Table 1

Sapfir-22DD differential pressure sensor, No 54

X, atm	*Y, mV*	$\overline{Y}, mV$	Δ, $\boldsymbol{mV}$
0	2000.000	2028.545	28.545
0.1	2838.000	2825.709	–12.292
0.2	3651.000	3622.872	–28.128
0.3	4430.000	4420.036	–9.964
0.4	5216.000	5217.200	1.200
0.5	6007.000	6014.364	7.364
0.6	6800.000	6811.528	11.528
0.7	7600.000	7608.691	8.691
0.8	8403.000	8405.854	2.854
0.9	9205.000	9203.020	–1.981
1.0	10008.000	10000.182	–7.817

The coefficients of the CF of the “Sapphire-22DD” differential measuring device using the measurements tested in Table 1 are determined as a_1= .202854E+04; a_2= .797164E+04; a_3= .000000E+00; a_4= .000000E+00. As we see, the coefficients of the third and fourth terms of the polynomial are zero, and it is experimentally confirmed that the CC is taken as a square trinomial.

D = .243735E + 03 is the variance of the measurement result; F = .112206E + 01 is the value of the function F in the estimation of the coefficients of the polynomial.

5. Conclusion

The proposed differential measurement method and the developed hybrid test algorithm surpass the known methods with their simplicity, easy application and high measurement accuracy. Thus, the nonlinear identification of CF in the MS is carried out by simple additive, multiplicative tests and their combination, there is no need to determine the parameters (coefficients) of CF, the adequacy of measurement results depends on the accuracy class of the selected tests and the approximation step of CC and is carried out in the form of a square trinomial.

References

[1] H.Ə.Məmmədov, Ç.M. Hacıyev, Ölçmə nəticələrinin emal metodları, Bakı, «Sabah», (2008) 275 s. [In Azerbaijani: H.A. Mamedov, Ch.M. Hajiyev, Methods of Processing Measurement Results, Baku, Sabah]

[2] M.M. İsayev, Ölçmə sistemlərinin dəqiqliyinin yüksəldilməsinin alqoritmik-test üsulları, Bakı, «Elm» nəşriyyatı, (2018) 206 s. [In Azerbaijani: M.M. Isayev, Algorithmic-test Methods for Improving the Accuracy of Measurement Systems, Baku, Elm]

[3] Э.М. Бромберг, К.Л. Куликовский, Тестовые методы повышения точности измерений, Москва, Энергия, (1987) 176 с. [In Russian: EM. Bromberg, K.L. Kulikovsky, Test Methods for Improving Measurement Accuracy, Moscow, Energiya]

[4] Г. Корн, Т. Корн, Справочник по математике для научных работников и инженеров, Москва, Наука, (1978) 720 с. [In Russian: G. Korn, T. Korn, Handbook of Mathematics for Scientists and Engineers, Moscow, Nauka]

[5] В.А. Ильин, Э.Г. Позняк. Линейная алгебра: Учеб.: Для вузов, 6-е изд., стер., Москва, ФИЗМАТЛИТ, (2007) 280 с. [In Russian: V.A. Ilyin, E.G. Poznyak. Linear Algebra: Textbook: For universities, 6th ed., Moscow, FIZMATLIT]