Informatics and Control Problems 43 No.1 (2023)
journal homepage: www.icp.az

The substantiation of the application of the integral criterion at the data processing

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ARTICLE INFO

Article history:
Received 19.09.2022
Received in revised form 11.10.2022
Accepted 18.10.2022
Available online 05.04.2023

Keywords:
Mathematical model
Identification
Calibration
Aircraft
Non-smooth optimization
Barrier function

ABSTRACT

The problem of online calibration of navigational instruments is considered. Calibration of devices implies minimizing the deviations between the real values and the measurement data. In the problem under investigation, the trajectory of motion is traced with GPS, which is taken as the real values of the trajectory coordinates. The accelerometer and gyroscope measurement data can also be used to calculate the trajectory. It is required by calibrating the sensors to achieve matching of these values, with the proximity of trajectories implying a uniform metric. It is proved in the article that the integral metric can be used in solving the problem of sensor calibration.

1. Introduction

Controlling an aircraft requires certain information about its condition. Some of this information is taken from sensors installed on the aircraft. This paper explores the possibility of eliminating systematic errors in the data obtained from the sensors.

Sensors normally give electrical signals. These signals are then digitized in accordance with the purpose of the sensor. In the process of digitization, different dependencies can be used. For instance, load cells convert volts to quantities expressed in m/s^2. Sensor manufacturers usually set the conversion function when testing the sensors.

However, over time, due to the deterioration of sensors operating on electromechanical principles, the application of the conversion function set by the manufacturing plant leads to errors, resulting in significant deviations from the real values. Therefore, it is necessary to adjust the conversion function in order to eliminate errors in the processing of flight data from the sensors. The process of adjusting the conversion functions in such a way as to ensure adequacy is called calibration.

In this paper, we investigate the possibility of adjusting the data obtained from sensors installed on board unmanned aerial vehicles using the additional information about the flight and propose a solution concept, as well as an appropriate solution algorithm. In addition, the correctness of the solution concept is proven mathematically.

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www.icp.az/2023/1-10.pdf https://doi.org/10.54381/icp.2023.1.10

2. Problem statement

The main navigation devices used in aviation are accelerometers and gyroscopes. Accelerometers measure aircraft's loads. By load, we mean the ratio of the sum of all aerodynamic forces acting on the aircraft, with the exception of gravity, and the engine thrust to gravity [1, P.47]. Load can be expressed as follows:

$$n = \frac{P + R_A}{mg},$$

where P is the thrust of the aircraft engines, R_A is the sum of the aerodynamic forces acting on the aircraft, m is its mass, and g is the gravitational acceleration. Gyroscopes provide orientation angles that determine the position of the aircraft in space relative to the earth. To interpret them, we first introduce the coordinate systems being used.

Let us denote the fixed-in-the-earth coordinate system by $O_g x_g y_g z_g$ and the co-ordinate system fixed to the aircraft by $Oxyz$. For clarity, it is assumed that the Oz_g axis is directed vertically upward in the considered point, and the $O_g x_g$ and $O_g y_g$ axes are directed so that the $O_g x_g z_g$ plane is perpendicular to $O_g z_g$, forming a right-handed coordinate system. The $Oxyz$ coordinate system fixed to the aircraft is introduced such that when the aircraft stands on the ground, the Ox, Oy and Oz axes are parallel to the $O_g x_g$, $O_g y_g$ and $O_g z_g$ axes of the fixed-in-the-earth $O_g x_g y_g z_g$ coordinate system, respectively.

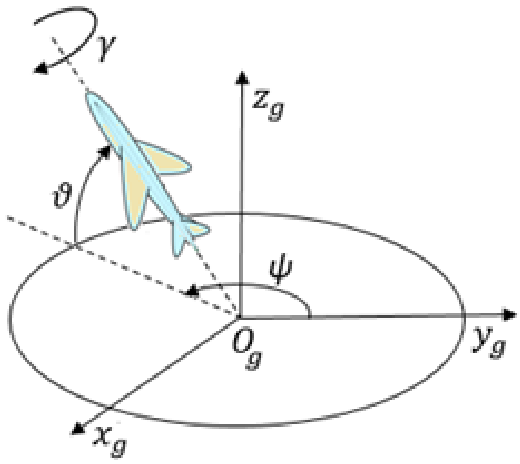

Fig.1. Orientation angles

The orientation of the aircraft relative to the ground is determined by the ψ , ϑ , γ angles [1, P.431; 2 P.9]. Here ψ is called the yaw angle – the angle between the $O_g x_g$ axis and the projection of the Ox axis on the horizontal $O_g x_g z_g$ plane; ϑ is the pitch angle – the angle between the Ox axis and the horizontal $O_g x_g z_g$ plane; γ is the roll angle – the angle between the Oz axis and the horizontal $O_g x_g z_g$ plane (Fig. 1).

Let $p_x(t)$, $p_y(t)$ and $p_z(t)$ denote the values of the load $\boldsymbol{n}$ measured by the accelerometer in volts at each instant of time *t*. Suppose that the functions $k_x(p_x)$, $k_y(p_y)$, $k_z(p_z)$ are conversion functions that allow calculating the physical values of the loads:

$$\begin{cases} n_x = k_x(p_x), \\ n_y = k_y(p_y), \\ n_z = k_z(p_z). \end{cases} \quad (1)$$

Given that the spring mechanisms of the accelerometer lose their elasticity over time, we can assume that the values of the loads change proportionally to a certain coefficient. On the other hand, changes in the sensitivity threshold can lead to systematic errors in the accelerometer measurements. Combining these two cases, we can assume that by applying formulas (1) to the quantities p_x, p_y, p_z, some intermediate values $\hat{n}_x$, $\hat{n}_y$ and $\hat{n}_z$ for loads are determined, which in turn are bound with real values of n_x, n_y, n_z of the loads by the following linear dependence formulas:

$$\begin{cases} n_x = \alpha_1 \hat{n}_x + \beta_1, \\ n_y = \alpha_2 \hat{n}_y + \beta_2, \\ n_z = \alpha_3 \hat{n}_z + \beta_3, \end{cases} \tag{2}$$

where $\alpha_1, \alpha_2, \dots, \beta_3$ are certain coefficients. Finding these coefficients determines the notion of the sensor adjustment. Thus, additional information is required to find them. The proposed approach to finding the coefficients is based on the fact that the GPS measuring device operates on board the aircraft during a certain brief period of time T from the moment of flight $t = 0$, and on the basis of its data, it is possible to determine the speed of the aircraft during that period. Thus, GPS data related to the initial period of flight can be considered sufficiently reliable.

Loads are measured relative to the coordinate system fixed to the aircraft. For a comparison with GPS data, they must be expressed in the coordinate system relative to the earth.

Let us denote the components of the aircraft's load vector in the $O_g x_g y_g z_g$ coordinate system respectively by n_{gx}, n_{gy}, n_{gz}. Then the conversion formulas in the matrix notation will be as follows [3, p.23-25]:

$$\begin{pmatrix} n_{gx} \\ n_{gy} \\ n_{gz} \end{pmatrix} = A_R(\psi) A_T(\vartheta) A_K(\gamma) \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix},$$

where

$$A_K(\gamma) \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}, \quad A_T(\vartheta) \equiv \begin{pmatrix} \cos\vartheta & -\sin\vartheta & 0 \\ \sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$A_R(\psi) \equiv \begin{pmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{pmatrix}.$$

Depending on the nature of the loads [1], it is possible to calculate the acceleration vector $\boldsymbol{a}(t) = (a_x, a_y, a_z)$ that depends on the forces acting on the aircraft during flight (gravity, aerodynamic drag, engine thrust):

$$\begin{cases} a_x = g \cdot n_{gx}, \\ a_y = g \cdot (n_{gy} - 1), \\ a_z = g \cdot n_{gz}, \end{cases}$$

where g is the gravitational acceleration. Let us denote by $\boldsymbol{V}(t) = (V_x(t), V_y(t), V_z(t))$ the velocity vector determined on the basis of the GPS device data. The problem of determining conversion functions (2) for the sensor adjustment implies finding coefficients $\alpha_1, \alpha_2, \dots, \beta_3$ such that for each $t \in [0,1]$, the functional

$$J(\alpha_1, \alpha_2, \dots, \beta_3) \equiv \left\| V(t) - \int_0^t a(\tau) d\tau \right\| \tag{3}$$

gets the minimum value.

In accordance with the logic of the problem, approximation (3) should be regular. In other words, the approximation should be considered in the space of C-continuous functions. However, in this case, the problem is non-smooth and it becomes necessary to apply one of non-smooth optimization methods [4, 5]. In practice, the approximation problem is often considered with a quadratic functional. The main reason for this is the availability of easily applicable algorithms [6] for minimizing the quadratic functional.

Indeed, let us consider the problem of finding the minimum of functional (3) when it is expressed by the norm L_2 (quadratically integrable functions). Denote by (a_{ij}), $i,j=1,2,3$ the elements of the matrix $A_R(\psi)A_T(\vartheta)A_K(\gamma)$:

$$a_{11}=\cos\psi\cos\vartheta\,,\, a_{12}=\cos\psi\sin\vartheta\sin\gamma-\sin\psi\cos\gamma\,,\, a_{13}=\cos\psi\sin\vartheta\cos\gamma+\sin\psi\sin\gamma\,,\, a_{21}=\sin\psi\cos\vartheta\,,\, a_{22}=\sin\psi\sin\vartheta\sin\gamma+\cos\psi\cos\gamma\,,\, a_{23}=\sin\psi\sin\vartheta\cos\gamma-\cos\psi\sin\gamma\,,\, a_{31}=-\sin\vartheta\,,\, a_{32}=\cos\vartheta\sin\gamma\,,\, a_{33}=\cos\vartheta\cos\gamma.$$

Then functional (3) can be written as follows:

$$J(\alpha_1,\alpha_2,\dots,\beta_3)=\sqrt{\int_0^T\left|V(t)-\int_0^t a(\tau)d\tau\right|^2 dt}=$$

$$\left\{\int_0^T\left[\left(V_x(t)-\int_0^t a_x(\tau)d\tau\right)^2+\left(V_y(t)-\int_0^t a_y(\tau)d\tau\right)^2+\left(V_z(t)-\int_0^t a_z(\tau)d\tau\right)^2\right]dt\right\}^{\frac{1}{2}}=$$

$$\left\{\int_0^T\left(\left[g\int_0^t\left(a_{11}(\alpha_1\hat{n}_x+\beta_1)+a_{12}\left(\alpha_2\hat{n}_y+\beta_2\right)+a_{13}(\alpha_3\hat{n}_z+\beta_3)\right)d\tau-V_x(t)\right]^2+\right.\right.$$

$$+\left[g\int_0^t\left(a_{21}(\alpha_1\hat{n}_x+\beta_1)+a_{22}\left(\alpha_2\hat{n}_y+\beta_2\right)+a_{23}(\alpha_3\hat{n}_z+\beta_3)-1\right)d\tau-V_y(t)\right]^2+$$

$$\left.\left.+\left[g\int_0^t\left(a_{31}(\alpha_1\hat{n}_x+\beta_1)+a_{32}\left(\alpha_2\hat{n}_y+\beta_2\right)+a_{33}(\alpha_3\hat{n}_z+\beta_3)\right)d\tau-V_z(t)\right]^2\right)\right\}^{\frac{1}{2}}.$$

To find the minimum of the obtained function, we calculate the partial derivatives $\frac{\partial J}{\partial\alpha_k}$ and $\frac{\partial J}{\partial\beta_k}$ and equate them to zero. Thus, the calculation of the minimum of the functional $J(\alpha_1,\alpha_2,\dots,\beta_3)$ comes down to solving a system of linear algebraic equations for the variables $\alpha_1,\alpha_2,\dots,\beta_3$ and is easily solved.

As we can see, in the previous paragraphs, to find the coefficients $\alpha_1,\alpha_2,\dots,\beta_3$, the norm expressing functional (3) was understood in the L_2 sense. Let us prove mathematically that in a number of natural conditions put forward for the original problem, the problem considered with the quadratic functional can be conformed to the solution of the problem considered with the regular functional.

3. Substantiation of the application of the integral criterion

As mentioned above, functional (3) must be minimized in the regular metric. However, when solving the problem, the L_2 metric was used instead of the regular metric. To substantiate the use of the L_2 metric, it is necessary to conform the L_2 metric to the regular metric. In the general case, one can easily construct an example where the norm of the function in the L_2 sense, being arbitrarily small, can remain larger than the constant defined in the C metric.

On the other hand, the function participating in functional (3) is subject to certain restrictions, since it reflects the speed of the aircraft. Thus, the acceleration that the aircraft can get depends on the power of its engines and is always limited to a certain constant. Within the indicated restrictive condition, it can be shown that the sequence minimizing functional (3) in the L_2 sense is also fundamental in the regular metric. In fact, the value of the regular metric is bounded from above by 2/3 order of the value of the L_2 metric. A mathematical proof of this is given in the following theorem.

Theorem. Denote by Ω the set of functions satisfying the conditions $u \in W_2^1[0,1]$ and $|u'(t)| \leq K$. Let us prove that for any $u \in \Omega$

$$\|u\|_{C[0,1]}^{\frac{3}{2}} \leq \sqrt{3K}\|u\|_{L_2[0,1]}, \tag{4}$$

where

$$\|u\|_{C[0,1]} = \max_{t\in[0,1]}|u(t)|,$$

$$\|u\|_{L_2[0,1]} = \sqrt{\int_0^1 |u(t)|^2 dt}.$$

Proof. Suppose that

$$\|u\|_{C[0,1]} = u_* \tag{5}$$

and this value is realized at a point $t_* \in [0,1]$, i.e. $|u(t_*)| = u_*$.

According to the definition of norms $\|u\|_{C[0,1]}$ and $\|u\|_{L_2[0,1]}$, for any $u \in \Omega$, $(-u) \in \Omega$ and $\|-u\|_{C[0,1]} = \|u\|_{C[0,1]}$ and $\|-u\|_{L_2[0,1]} = \|u\|_{L_2[0,1]}$. Therefore, we can assume that the function $u(t)$ passes through the point $B(t_*, u_*)$, and $u(t_*) = u_*$ (Fig. 2). Denote by $A(t_1, 0)$ and $C(t_2, 0)$ the points of intersection with the Ot axis of the straight lines that pass through point B and whose equations are written in the form $u = u_* \pm K(t - t_*)$. Let us construct such a function

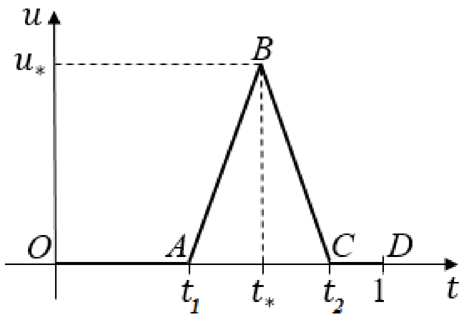

Fig.2. Construction of the barrier function

$$\tilde{u}(t) = \begin{cases} 0, & t < t_1, \\ u_* + K(t - t_*), & t_1 \leq t \leq t_*, \\ u_* - K(t - t_*), & t_* \leq t \leq t_2, \\ 0, & t > t_1. \end{cases}$$

and consider its restriction to $[0,1]$. From the condition $|u'(t)| \le K$, it is clear that the graph of each function $u \in \Omega$ passing through the point $B(t_*, u_*)$ cannot be lower than the graph of the function $\tilde{u}(t)$, in other words, $u(t) \ge \tilde{u}(t)$ and

$$\|\tilde{u}(t)\|_{L_2[0,1]} \le \|u(t)\|_{L_2[0,1]} \tag{6}$$

The function $\tilde{u}(t)$ constructed in this fashion is called a barrier function. The norm $L_2[0,1]$ of barrier functions corresponding to all functions with $\|u\|_{C[0,1]} = u_*$ takes different values depending on which point of the section the point t_* is located. $\|u(t)\|_{L_2[0,1]}$ takes the minimum value when the point t_* is an endpoint of the section $[0,1]$: $t_* = 0$ or $t_* = 1$. This corresponds to one of the following two cases for the function $\tilde{u}(t)$:

I. $$t_* = 0, \frac{u_*}{K} < 1 \text{ və } \tilde{u}(t) = \begin{cases} u_* - Kt, & 0 \le t < \frac{u_*}{K}, \\ 0, & \frac{u_*}{K} \le t \le 1. \end{cases}$$

II. $$t_* = 1, \frac{u_*}{K} < 1 \text{ və } \tilde{u}(t) = \begin{cases} 0, & 0 \le t < 1 - \frac{u_*}{K}, \\ u_* + K(t-1), & 1 - \frac{u_*}{K} \le t \le 1. \end{cases}$$

First, we consider the first case – according to the definition of the norm,

$$\|\tilde{u}(t)\|_{L_2[0,1]} = \sqrt{\int_0^1 |\tilde{u}(t)|^2 dt}\,.$$

Taking into account that $\int_0^1 |\tilde{u}(t)|^2 dt = \int_0^{\frac{u_*}{K}} |\tilde{u}(t)|^2 dt + \int_{\frac{u_*}{K}}^1 |\tilde{u}(t)|^2 dt$ and $\tilde{u}(t) = 0$ on the section $[0, 1]$,

$$\|\tilde{u}(t)\|_{L_2[0,1]} = \sqrt{\int_0^{\frac{u_*}{K}} (u_* - Kt)^2 dt} = \frac{1}{\sqrt{3K}} (u_*)^{\frac{3}{2}}\,.$$

Finally, from equality (5)

$$\|\tilde{u}(t)\|_{L_2[0,1]} = \frac{1}{\sqrt{3K}} \|u\|_{C[0,1]}^{\frac{3}{2}}. \tag{7}$$

A similar result is obtained in the second case. Given equation (7) in (6), we obtain inequality (4).

4. Conclusion

The problem of sensor calibration based on the proximity of the trajectory tracked with GPS and the data of navigational instruments in a uniform metric is investigated. Normally, effective algorithms based on minimization of data divergence in the integral metric are used to solve such problems. It is proved in this article that under certain natural conditions of problems under investigation, integral metric can act as a major of the uniform metric. Thus, a substantiation of applying the method of minimization of standard deviation in the problem under investigation is given. This approach can be also applied to other identification and modeling problems instead of the uniform metric.

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