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Generalization of the Kalman equation to a linear-quadratic optimal control problem

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ABSTRACT

In a Hilbert space, a linear-quadratic optimal control problem with a fixed left end and a movable right end is considered. At the right end of the time interval, a linear programming problem is formulated, the solution of which implicitly determines the terminal condition. A saddle point approach is proposed, which is reduced to calculating the saddle point of the Lagrange function. It is based on saddle point inequalities for both groups of variables: direct and dual. These inequalities are sufficient conditions for optimality. A control synthesis is constructed that implements feedback in the presence of constraints on controls as a convex closed set. This is a new result, since in the classical case in the theory of a linear controller (Kalman equation) a similar result is proven only in the absence of constraints on controls.

1. Introduction

It is known that the maximum principle is one of the most powerful tools for solving optimal control problems. However, the maximum principle, from the point of view of developing the theory of methods for solving optimal control problems, is only a necessary condition for optimality. This means that the solution found using the maximum principle may or may not be the solution to the original problem. Then, the computational method cannot be considered justified from the point of view of demonstrative calculations. In practice, the experience and intuition of experts are often relied upon to justify the obtained solution. However, this kind of expertise is subjective and is not demonstrative.

Sufficient optimality conditions seem to be a natural way out of the situation. The main ideas of this approach were discussed in [1], but they were not clearly substantiated there; it was only assumed that a linear regulator (feedback) could be synthesized based on the saddle point method. In this paper, it is proved that the feedback mechanism is entirely “embedded” in the computational technology of the saddle point gradient method [2–13]. In fact, these processes are interrelated. Otherwise, for variational inequalities see [14].

The validity of the approach is based on proven theorems, which are based on duality theory. All theorems work on classes of methods and for classes of problems. The convergence of computational processes is guaranteed for all components of the solution, namely, for controls (weak), for phase and dual trajectories (strong), and for terminal variables convergence is also strong.

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2. Problem statement

In this paper, we consider the problem of terminal control with linear dynamics at a finite time interval:

$$\frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t), \quad t_0 \le t \le t_1,$$

where $D(t), B(t)$ are functional matrices. The left end $x(t_0)$ of the phase variable is fixed and equal to x_0; the right end $x(t_1)$ is movable within the convex polyhedron $M = \{x_1 \in R^n |\ G_1 x_1 \le g_1\}$;$G_1$ is a given matrix of dimension $m \times n,\ m < n$ and vector g_1 is fixed. The control $u \in U \subset L_2^r\,[t_0, t_1]$, and U is a convex closed set. For any controls from U and given $x(t_0)$, the system of differential equations (1) generates trajectories $x(\cdot)$whose right ends describe the reachability set $X_1 \subset R^n$. The problem is considered in Hilbert space. The trajectory $x(\cdot)$ is an absolutely continuous function. The class of absolutely continuous functions will be denoted as $AC_2^n[t_0 . t_1]$.

By a solution of the differential system above will be understood as any pair $(x(t), u(t))$, identically satisfying the condition

$$x(t) = x(t_0) + \int_{t_0}^{t} \big(D(t)x(t) + B(t)u(t)\big)\, dt, \qquad t_0 \le t \le t_1.$$

In [15], Book 1, p. 443, it is shown that any control $u \in U$ in a linear differential system is associated with a unique trajectory satisfying this identity.

The minimized functional is represented by the sum of the integral and terminal components:

$$J(u) = \frac{1}{2}\int_{t_0}^{t_1} \langle Q_1(t)x(t), x(t)\rangle + \langle Q_2(t)u(t), u(t)\rangle\, dt + \langle Sx_1, x_1\rangle,$$

where S is a fixed positive semidefinite symmetric matrix; Q_1, Q_2 are continuous positive semidefinite symmetric matrices of dimensions $n \times n$ and $n \times r$, respectively.

Given the aforesaid, we formulate the terminal control problem in the form

$$\begin{cases} (x_1^*, x^*(\cdot), u^*(\cdot)) \in \\ Argmin\{J(u) \mid G_1 x_1 \le g_1, x_1 \in X_1 \subset R^n, \\ \frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t), x(t_0) = x_0, x(t_1) = x_1^*\,, \\ t \in [t_0 . t_1],\ \ x(\cdot) \in AC_2^n[t_0 . t_1], u(\cdot) \in U. \end{cases} \tag{1}$$

There are several types of sufficient conditions in optimal control theory. The best known of them are the V.F. Krotov sufficient conditions, based on the Hamilton–Jacobi inequalities [16], and V.V. Velichenko condition, based on the theory of the field of extremals [17]. In our case, we use sufficient conditions of the saddle point type, which follow from the duality theory and that we obtained in the case of linearization of convex and smooth problems of terminal control. They were called saddle point conditions, since they are generated by saddle point inequalities with respect to the primal and dual variables of the Lagrange function in problems of linear and convex programming.

In this problem, it is required to find a control $u^*(\cdot) \in U$ such that the corresponding trajectory $x^*(\cdot)$ connects the initial point x_0 with the terminal point $x^*(t_1) \in X_1$, while the objective functional takes a minimum value. In [15], Book 2, p. 653, it is proved that a solution $(x_1^*, x^*(\cdot), u^*(\cdot)) \in R^n \times AC_2^n[t_0 . t_1] \times U$ of this problem exists.

3. Dual problem

Problem (1), an analogue of the linear programming problem, is formulated in a Hilbert space. Drawing analogies between the formulations of optimization problems in finite-dimensional and functional spaces, we introduce the Lagrangian function for (1):

$$L\big(x_1, x(\cdot), u(\cdot); p_1, \psi(\cdot)\big) = + \int_{t_0}^{t_1} \langle Q_1(t)x^*(t), x(t)\rangle + \langle Q_2(t)u^*(t), u(t)\rangle \mathrm{dt}$$

$$+ \int_{t_0}^{t_1} \langle \psi(t), D(t)x(t) + B(t)u(t) - \frac{d}{dt}x(t)\rangle\, dt$$

$$+ \langle \mathrm{S}x_1^*, x_1\rangle + \langle p_1, G_1x_1 - g_1\rangle \qquad (2)$$

for all $(x_1^*, x^*(\cdot), u^*(\cdot)) \in R^n \times AC_2^n[t_0.t_1] \times U$, $(p_1, \psi(\cdot) \in R_+^m \times \Psi_2^n[t_0.t_1]$. Here, $\Psi_2^n[t_0.t_1]$ is the linear variety of absolutely continuous functions from the dual space. According to the Kuhn–Tucker theorem, a saddle point $(x_1^*, x^*(\cdot), u^*(\cdot); p_1^*, \psi^*(\cdot))$ of the Lagrange function is formed by primal $(x_1^*, x^*(\cdot), u^*(\cdot))$, and dual $(p_1^*, \psi^*(\cdot))$ variables.

By definition, a saddle point of L satisfies the system of inequalities

$$L\big(x_1^*, x^*(\cdot), u^*(\cdot); p_1, \psi(\cdot)\big) \le L\big(x_1^*, x^*(\cdot), u^*(\cdot); p_1^*, \psi^*(\cdot)\big) \le L(x_1, x(\cdot), u(\cdot); p_1^*, \psi^*(\cdot)) \qquad (3')$$

We introduce the concept of the dual Lagrangian and show that it generates the dual problem.

$$L^T\big(x_1, x(\cdot), u(\cdot); p_1, \psi_1, \psi(\cdot)\big) =$$

$$+ \int_{t_0}^{t_1} \langle Q_1(t)x^*(t) + D^T(t)\psi(t) + \frac{d}{dt}\psi(t), x(t)\rangle\, dt$$

$$+ \int_{t_0}^{t_1} \langle Q_2(t)u^*(t) + B^T(t)\psi(t), u(t)\rangle dt$$

$$+ \langle \mathrm{S}x_1^* + G_1^T p_1 - \psi_1, x_1\rangle + \langle p_1, -g_1\rangle + \langle \psi_0, x_0\rangle \qquad (3'')$$

where $\psi_0 = \psi(t_0)$, $\psi_1 = \psi(t_1)$. Both Lagrangians (3') and (3'') have the same set of saddle points.

The saddle point system allows one to move on to the dual problem

$$\begin{cases} (p_1^*, \psi_1, \psi^*(\cdot)) \in \\ Argmax\Big\{ \int_{t_0}^{t_1} \langle \psi(t), B(t)u^*(t)\rangle dt + \langle p_1, -g_1\rangle \mid \\ \frac{d}{dt}\psi(t) + D^T(t)\psi(t) = -Q_1(t)x^*(t), x(t_0) = x_0, x(t_1) = x_1^*, \\ \psi_1 = Sx_1^* + G_1^T p_1,\ (p_1, \psi(\cdot)) \in R_+^m \times \Psi_2^n[t_0.t_1]\}, \\ \int_{t_0}^{t_1} \langle Q_2(t)u^*(t) + B^T(t)\psi(t), u(t)\rangle dt. \end{cases} \qquad (4)$$

4. Saddle point sufficient conditions for optimality

Considering together the left inequality of the source saddle point system and the right inequality of the dual saddle point system, we arrive at the following problem:

$$\frac{d}{dt}x^*(t) = D(t)x^*(t) + B(t)u^*(t),\ x^*(t_0) = x_0,$$

$$\langle p_1 - p_1^*, G_1x_1^* - g_1\rangle \le 0,\quad p_1 \in R_+^m,$$

$$\frac{d}{dt}\psi^*(t) + D^T(t)\psi^*(t) = -Q_1(t)x^*(t),$$

$$\psi_1^* = \ \mathrm{S}x_1^* + G_1^T p_1^*,$$

$$\int_{t_0}^{t_1} \langle Q_2(t)u^*(t) + B^T(t)\psi^*(t), u^*(t) - u(t)\rangle dt \le 0,$$

$$u(\cdot) \in U. \tag{5}$$

This differential system is a saddle-type sufficient condition for optimality.

Comparing this system with the maximum principle, we can see that the main and only difference is in the last inequality. In our convex case, this inequality is a variational inequality with respect to the set in a Hilbert space. In the smooth case of the maximum principle, we will have a parametric family of finite-dimensional variational inequalities depending on the time parameter $t \in [t_0 . t_1]$.

5. Method for solving and proof of its convergence

The formulas of the controlled iterative extragradient method are as follows:

(1) predictive half-step ($\alpha > 0$):

$$\frac{d}{dt}x^k(t) = D(t)x^k(t) + B(t)u^k(t),\ x^k(t_0) = x_0,\ \overline{p}_1^k = \pi_+\left(p_1^k + \alpha\left(G_1 x_1^k - g_1\right)\right),$$

$$\frac{d}{dt}\psi^k(t) + D^T(t)\psi^k(t) = -Q_1(t)x^k(t),$$

$$\psi_1^k = Sx_1^k + G_1^T p_1^k,$$

$$\overline{u}^k(t) = \pi_U(u^k(t) - \alpha(Q_2(t)u^k(t) + B^T(t)\psi^k(t)));$$

(2) the main half-step:

$$\frac{d}{dt}\overline{x}^k(t) = D(t)\overline{x}^k(t) + B(t)\overline{u}^k(t),\ \ \overline{x}^k(t_0) = x_0,\ p_1^{k+1} = \pi_+\left(p_1^k + \alpha\left(G_1\overline{x}_1^k - g_1\right)\right),$$

$$\frac{d}{dt}\overline{\psi}^k(t) + D^T(t)\overline{\psi}^k(t) = -Q_1(t)\overline{x}^k(t),$$

$$\overline{\psi}_1^k = S\overline{x}_1^k + G_1^T\overline{p}_1^k,$$

$$u^{k+1}(t) = \pi_U\left(u^k(t) - \alpha(Q_2(t)\overline{u}^k(t) + \ B^T(t)\overline{\psi}^k(t))\right),\ k = 0,1,2,\dots \tag{6}$$

We have proven the convergence of the method to one of the solutions of the source problem.

Theorem (convergence). If the set of solutions $\left(x_1^*, x^*(\cdot), u^*(\cdot); p_1^*, \psi_1^*, \psi^*(\cdot)\right)$ *to problem* (5) *is not empty and belongs to space* $R^n \times AC_2^n[t_0 . t_1] \times U \times R_+^m \times R^n \times \Psi_2^n[t_0 . t_1]$ *then sequence* $\left(x_1^k, x^k(\cdot), u^k(\cdot); p_1^k, \psi_1^k, \psi^k(\cdot)\right)$, *generated by method* (6) *with step* α *chosen from some condition, contains a subsequence that converges to the solution of the problem. The convergence in controls is weak, and the convergence in all other variables is strong. Moreover, the sequence* $\left\{\left|p_1^k - p_1^*\right|^2 + \|u^k(\cdot) - u^*(\cdot)\|^2\right\}$ *monotonically decreases on* $R_+^m \times \mathrm{L}_2^r[t_0 . t_1]$.

6. Synthesis of control for linear differential system with constraints on control

Let us show how, based on the proposed theory, it is possible to construct (synthesize) control of the linear system from (1) in the presence of constraints on control. Let's turn to system (5). As already noted, the solution of this system is a saddle point. This means that, if x_0 and control $u(t)$ are known, then we can solve the differential equation in (5) and find the trajectory $x(t)$. Then we can calculate its value x_1 at the right end and find p_1 from the variational inequality. Having formed the transversality condition, we can solve the adjoint differential system and find the adjoint trajectory $\psi(t)$ with which we can form the normal to the support plane $\int_{t_0}^{t_1}\langle Q_2(t)u(t) + B^T(t)\psi(t), u(t)\rangle dt$ for the function set U. Then it remains only to find the point $u(\cdot) \in U$. that is the support point for our linear functional. If the initial data were optimal, then the found control coincides with the optimal one $u^*(t)$. In this case, the solution of (5) is found.

This system has a very simple algorithmic structure. In fact, from the first equation of the system of two variables, we must be expressed via the other and, using the variable obtained from the transversality condition, determine the terminal value for the adjoint differential equation; then solve this equation; and, using the found solution, form the normal to the plane on a convex closed controls set U and find the support point.

This substantive scheme is algorithmic in nature, so we will present it in the form of an algorithm. To this end, we will consider the following mappings (operators).

1. We introduce an operator $K_1(u) = x\big(t_1; u(t)\big)$ that assigns to each control $u \in U$ the right end x_1 of the trajectory $x(t; u(t))$. In [15], it was shown that the operator defined in this way, acting from a Hilbert space to a finite-dimensional one, is single-valued, linear, and bounded (continuous).

2. The transversality condition for each x_1 uniquely determines the terminal condition $\psi_1 = Sx_1 + G_1^T p_1$ for the adjoint differential equation. This equation generates the adjoint trajectory $\psi(t) = K_2(x) = \psi(x_1; t)$. Both mappings depend on parameters that coordinate the domains of definition of the operators so that the second is defined on the image of the first. In the method described in this paper, this is done automatically. Therefore, it seems natural to consider a superposition of the operators.

3. Now we introduce the operator $K_3(\psi) = VI(U; \psi)$, which associates each element $K_3(\psi)$ of the dual space with a solution of the last variational inequality (5). The vector $\psi(t)$ generates the normal of linear functional $Q_2(t)u^*(t) + B^T(t)\psi^*(t)$, defined on the control set U.

All three mappings depend on parameters that are coordinated by means of the terminal conditions in such a way that all mappings generate a superposition. This superposition is a mapping that transforms the set U into itself, and its fixed point is a solution of the original problem. It is easy to see that this system of mappings can be transformed into a closed-loop control

$$u(t) = K(t_1)x(t),$$

where $u(t) = K(t_1)x(t) = K_3\big(K_2(K_1(u))\big)$. Then (5) with feedback control takes the form

$$\frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t), \quad x(t_0) = x_0,$$
$$u(t) = K(t_1)x(t). \tag{7}$$

Accordingly, computational process (6) can be regarded as an iterative process of solving (7) with feedback control. The feedback can be interpreted as a gradient descent to a saddle point of the Lagrangian of the terminal control problem. In fact, the control synthesis is embedded in process (6).

7. Linear regulator theory. Control synthesis for a linear differential system: a classical approach

Let us note one important special case of this problem, which is known as the linear regulator theory [18, 19]. In this case, there are no constraints on the controls; i.e., $U \equiv L_2^r[t_0.t_1]$. Then, the problem (1) takes the form

$$\frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t), x(t_0) = x_0, x(t_1) = x_1^*,$$
$$(x_1^*, x^*(\cdot), u^*(\cdot)) \in$$
$$Argmin\left\{\frac{1}{2}\int_{t_0}^{t_1}\langle Q_1(t)x^*(t), x(t)\rangle + \langle Q_2(t)u^*(t), u(t)\rangle \mathrm{dt} + \frac{1}{2}\langle \mathrm{S}x_1^*, x_1\rangle \mid u(\cdot) \in L_2^r[t_0.t_1]\right\}.$$

The differential system (5) for this problem, which is a necessary and sufficient condition for optimality, takes the form of a system of differential and finite-dimensional equations. We transform this system as follows:

$$\frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t), \qquad x(t_0) = x_0,$$

$$\begin{gathered}\frac{d}{dt}\psi(t) + D^T(t)\psi(t) = -Q_1(t)x^*(t),\\ \psi_1 \;=\; Sx_1^* + Q_2(t)u\ \ (t) + B^T(t)\psi(t),\\ u(t) = -Q_2^{-1}(t)B^T(t)\psi(t).\end{gathered} \tag{8}$$

First, it is necessary to exclude the phase variable $\psi(t)$ from this system. To this end, in [18], it was assumed that there should be a relation of the type

$$\psi(t) = P(t)x(t), \tag{9}$$

where $P(t)$ is a continuously differentiable matrix function of the corresponding dimension. Since (8) contains a derivative of this function, which we will need in further reasoning, we differentiate (9):

$$\frac{d}{dt}\psi(t) = \frac{dP(t)}{dt}x(t) + \frac{dx(t)}{dt}P(t).$$

From this equality and the second equation in (8) we find

$$\frac{dP(t)}{dt}x(t) + \frac{dx(t)}{dt}P(t) = -Q_1(t)x(t) - D^T(t)P(t)x(t).$$

We substitute $\psi(t)$ from (9) into the last equality in (8), and then substitute the result $u(t)$ we found into the first equality in (8). Then we get

$$\begin{gathered}\frac{d}{dt}x(t) = D(t)x(t) - B(t)Q_2^{-1}(t)B^T(t)P(t)x(t),\\ x(t_0) = x_0.\end{gathered}$$

Eliminating $\frac{d}{dt}x(t)$ from the last two equalities, we obtain

$$\begin{gathered}\frac{dP(t)}{dt}x(t) + P(t)\big(D(t)x(t) - B(t)Q_2^{-1}(t)B^T(t)P(t)x(t)\big) =\\ = -Q_1(t)x(t) - D^T(t)P(t)x(t).\end{gathered}$$

Hence,

$$\left(\frac{dP(t)}{dt} + P(t)D(t) + D^T(t)P(t) - P(t)B(t)Q_2^{-1}(t)B^T(t)P(t) + Q_1(t)\right)x(t) = 0.$$

This equality must be satisfied for arbitrary $x(t)$; therefore, the factor in the large parentheses must be zero. Hence, we obtain the matrix Riccati equation with the boundary condition in the form of the third equality in (8):

$$\begin{gathered}\frac{dP(t)}{dt} = -P(t)D(t) - D^T(t)P(t) + P(t)B(t)Q_2^{-1}(t)B^T(t)P(t) - Q_1(t),\\ P(t_1) = S.\end{gathered}$$

From the equation, it is clear that the $n \times n$ matrix $P(t)$ must be symmetric and positive definite. From the last equality in (8) and (9) we have

$$u(t) = -Q_2^{-1}(t)B^T(t)P(t)x(t),$$

i.e., the matrix $K(t_1)$ from (7) has the structure

$$K(t_1) = -Q_2^{-1}(t)B^T(t)P(t).$$

8. Conclusion

In this article, we have considered a terminal control problem with a finite-dimensional boundary value problem at the right end of the time interval. The problem has a convex structure, which allows us to develop a theory of saddle point methods for solving terminal control problems within the duality theory.

Monotonic convergence of the computational process on the Cartesian product of direct and dual variables, as well as weak convergence in controls and strong convergence in phase and conjugate trajectories and in terminal variables of the boundary value problem have been proved. Based on the saddle point approach, control synthesis has been constructed, i.e., feedback under constraints on controls in the form of a convex closed set. This is a new result, since, in the classical case in the linear regulator theory, a similar statement is proved only in the absence of constraints on controls. The theory of the linear controller is based on the matrix Riccati equations, while the result obtained in this paper is based on the concept of a support function (mapping).

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